Chapter 14

ISO-, GENO-, AND HYPER-GRAND-UNIFICATIONS AND ISO-, GENO-, HYPER-COSMOLOGIES

14.1 ISO-, GENO-, AND HYPER-GRAND-UNIFICATION

14.1.1 The Role of Antimatter in Grand Unifications

As indicated earlier, no conclusive study on antimatter can be conducted without its consistent inclusion in *grand unifications* of gravitational [1–3] and electroweak interactions [4–7]. Vice versa, no grand unification can be considered scientifically valuable without the correct inclusion of antimatter because the latter has a profound impact in the very structure of a consistent grand unification.

All studies on grand unifications conducted until now have been essentially restricted to matter. When antimatter is included, the studies have to be enlarged to *two grand unifications,* one for matter and the other for antimatter with a correct anti-automorphic (or anti-isomorphic) interconnecting map.

Consequently, the inclusion of antimatter in grand unifications introduces severe restrictions on the admissible models, which restrictions are generally absent when antimatter is ignored and grand unifications are restricted to matter alone.

We shall, therefore, avoid the review of the very large number of structurally inconsistent grand unifications published since Einstein's times and leave to the interested reader their re-examination in light of the new advances of this volume.

An in depth study of grand unifications soon reveals the need of formulating antimatter at the purely classical level, the need for abandoning curvature, and the need for a geometric unification of special and general relativities as presented in preceding chapters. It is only at the level of these broader views on grand unifications that the isodual theory of antimatter emerges as inevitable.

Even though presented at the end of this monograph, the author initiated his studies on grand unification, constructed the needed broadening or modifications of pre-existing methods, and then achieved an invariant, axiomatically consistent grand unification.

This process requires it two decades of research before the publication of the first paper on grand unification, a lapse of time illustrating the complexity of the problem, as known in any case by the failure of the large number of preceding attempts.

The reader should be aware that, in this section, we shall exclusively study *closed-isolated systems of electroweak and gravitational interactions in vacuum* that are treatable via the Lie-isotopic branch of hadronic mechanics and its isodual. Interior problems, such as those inclusive of the origin of gravitation, require the broader Lie-admissible branch of hadronic mechanics and their treatment will be merely indicated at the end of this section for development by interested readers.

14.1.2 Axiomatic Incompatibilities of General Relativity and Electroweak Interactions

The preceding efforts for a grand unification of gauge theories of electroweak interactions and gravitation as described by general relativity are afflicted by the following axiomatic incompatibilities, first presented in Ref. [9] of 1997 (see also the related papers [10,11]):

(1) **Incompatibilities due to antimatter:** electroweak theories are *bona fide* relativistic field theories, thus characterizing antimatter via *negative-energy* solutions, while general relativity characterizes antimatter via *positive-definite* energy-momentum tensors. This first incompatibility renders manifestly inconsistent all attempts at grand unification known to this author.[1]

(2) **Incompatibilities due to curvature:** electroweak theories are essentially flat theories since they are formulated via *Minkowskian* axioms, while general relativity is centrally dependent on curvature since it is based on *Riemannian* axioms. This second incompatibility is another, independent, primary origin of the failure of the vast number of attempts at grand unification existing in the literature and carries profound implications, such as the extension to grand unification of the theorems of catastrophic inconsistencies of Section 1.4.

[1]The indication of grand unifications inclusive of antimatter would be greatly appreciated.

(3) **Incompatibilities due to spacetime symmetries:** electroweak interactions are based on the axioms of special relativity, thus verifying the fundamental *Poincaré symmetry* $P(3.1)$, while such a basic symmetry is absent in general relativity and is replaced by a generic covariance. This third incompatibility has additional profound implications for any consistent grand unification because either one abandons the basic symmetries of electroweak interactions in favor of an unknown covariance, or one abandons general relativity for a new theory admitting a universal symmetry.

(4) **Incompatibilities due to the lack of a Minkowskian limit of general relativity:** as it is well known [1–3], general relativity admits a well defined *Euclidean* limit under PPN approximation, but one century of studies have failed to identify a corresponding well defined *Minkowskian* limit. On the other side, electroweak interactions [4–7] are formulated on a Minkowski spacetime. This fourth incompatibility of the two interactions then emerges in a number of aspects, such as irreconcilable ambiguities in the identification of total conservation laws of grand unifications when inclusive of gravitational interactions.

(5) **Incompatibilities due to the nonunitary character of quantum gravity:** as it is also well known, electroweak theories are *operator* field theories with a *unitary* structure, thus having invariant prediction of numerical values permitting meaningful experimental verifications. By comparison, all quantum formulations of general relativity (see, e.g. Ref. [8] and references quoted therein) have a *nonunitary* structure. Besides evident, additional, independent inconsistencies in attempting to combine unitary and nonunitary theories, any attempt of grand unification along contemporary views in general relativity and quantum gravity is afflicted by the theorems of catastrophic inconsistencies of Section 1.4.

It is evident that no significant advance can be achieved in grand unifications without, firstly, a serious addressing of these inconsistencies and, secondly, without their resolution.

Recall that the theory of electromagnetic interactions, when (and only when) restricted to the *vacuum*[2], has a majestic mathematical and physical consistency that eventually propagated to unified theories of electromagnetic and weak interactions.

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[2] It is well known by expert, but rarely spoken, that Maxwell's equations have no real physical value for the treatment of electromagnetism within physical media for countless reasons, some of which have been treated in Chapter 1. As an illustration, only to locally varying character of electromagnetic waves within physical media requires a radical revision of electromagnetism in the arena considered as a condition to pass from academic politics to real science.

The view adopted in this monograph, identifiable in more details only now, is that, rather than abandoning the majestic beauty of electroweak theories, we abandon instead the popular views on gravitation of the 20-th century due to their catastrophic inconsistencies and, as a condition to achieve a consistent grand unification, we reconstruct gravitational theories in such a way to have *the same abstract axioms* of electroweak theories.

14.1.3 Resolution of the Incompatibilities via Isotopies and Isodualities

In this chapter we present a resolution of the above incompatibilities first achieved by Santilli in Refs. [9] of 1997 (see also Refs. [10,11] following a number of rather complex and diversified scientific journeys that can be outlined as follows:

(A) Isotopies. The scientific journey to achieve a consistent grand unification started in 1978 with memoirs [12,13] for the classical and operator isotopies. A baffling aspect in the inclusion of gravity in unified gauge theories is their geometric incompatibility.

The view that motivated Refs. [12,13] is that the difficulties experienced in achieving a consistent grand unification are primarily due to *insufficiencies in their mathematical treatment*.

Stated in plain language, the view here considered is that, due to the complexity of the problem, the achievement of an axiomatic compatibility between gravitation and electroweak interactions requires a basically new mathematics, that is, basically new numbers, new spaces, new symmetries, etc.

Following first the verification of the lack of existence in the literature of a mathematics permitting the desired consistent grand unification, and following numerous attempts, the *only* possible new mathematics resulted to be that permitted by the *isotopies* as first proposed in Refs. [12,13], namely, a generalization of the conventional trivial unit +1 of electroweak theories into the most general possible, positive-definite unit with an unrestricted functional dependence on local variables, called *Santilli's isounit*,

$$I = +1 > 0 \quad \rightarrow \quad \hat{I} = \hat{I}^{\dagger} = I(x, v, \psi, \partial\psi, \dots) > 0, \tag{14.1.1}$$

and consequential compatible reconstruction of all main branches of mathematics.

The uniqueness of the isotopies is due to the fact that, *whether conventional or generalized, the unit is the basic invariant of any theory.* Therefore, the use of the unit for the generalization of pre-existing methods guarantees the preservation of the invariance so crucial for physical consistency (Sections 1.5.2 and 1.5.3).

Another aspect that illustrates the uniqueness of the isotopies for grand unifications is that *the positive-definiteness of the isounit guarantees the preservation of the abstract axioms of electroweak theories,* thus assuring axiomatic consistency of grand unification from the very beginning.

The general lines on isotopies presented in memoirs [12,13] of 1978 were then followed by laborious studies that reached mathematical and physical maturity only in memoir [14] of 1996, as outlined in Chapter 3 (see monographs [15] for a comprehensive presentation).

(B) Isodualities. The achievement of an axiomatically consistent grand unification for *matter* constitutes only *half* of the solution because, as stressed in Section 14.1.1, no grand unification can be considered physically significant without the consistent inclusion of antimatter.

The incompatibility of electroweak theories and general relativity for antimatter identified in Section 14.1.2 is only the symptom of deeper compatibility problems. As now familiar from the studies presented in this monograph, matter is treated at *all* levels, from Newtonian to electroweak theories, while antimatter is treated only at the level of *second quantization.*

Since there are serious indications that half of the universe could well be made up of antimatter (see Section 14.2), it is evident that a more effective theory of antimatter must apply at *all* levels.

Until such a scientific imbalance is resolved, any attempt at a grand unification can well prove to be futile.

Recall that charge conjugation in quantum mechanics is an *anti-automorphic map.* As a result, no classical theory of antimatter can possibly be axiomatically consistent via the mere change of the sign of the charge, because it must be an anti-automorphic (or, more generally, anti-isomorphic) image of that of matter in *all* aspects, including numbers, spaces, symmetries, etc.

The resolution of the above imbalance required a second laborious scientific journey that initiated with the proposal of the *isodual map* in memoirs [16] of 1985, here expressed for an arbitrary quantity

$$Q(x, v, \psi, \dots) \quad \rightarrow Q^d = -Q^\dagger(-x^\dagger, -v^\dagger, -\psi^\dagger, -\partial\psi^\dagger, \dots), \tag{14.1.2}$$

proposal that was followed by various studies whose mathematical and physical maturity was only reached years later in memoir [14] of 1996, as reported in Chapters 2 and 3 (see also monographs [15] for a more general presentation).

To illustrate the difficulties, it is appropriate here to note that, following the presentation in papers [16] of 1985 of the main mathematical ideas, it took the author *nine years* before publishing their application to antimatter in paper [17] of 1994.

We are here referring to the original proposal of Refs. [16,17] of mapping isounit (14.1.1) for matter into an *negative-definite* nonsingular arbitrary unit, known today as *Santilli's isodual isounits*,

$$\hat{I}(x, \psi, \partial\psi, \dots) > 0 \quad \rightarrow \quad \hat{I}^d = -\hat{I}^\dagger(-x^\dagger, -\psi^\dagger, -\partial\psi^\dagger, \dots) < 0 \qquad (14.1.3)$$

and its use for the characterization of antimatter at all levels, from Newtonian mechanics to second quantization.

The uniqueness of the isodual representation is given by the fact that isodualities are the *only* known liftings permitting the construction of a mathematic that is anti-isomorphic to the conventional (or isotopic) mathematics, as necessary for a consistent representation of antimatter at all levels, while preserving the crucial invariance needed to avoid catastrophic inconsistencies.

(C) Poincaré-Santilli isosymmetry and its isoduals. The scientific journeys on isotopies and isodualities were only intended as pre-requisites for the construction of the *universal symmetry of gravitation for matter and, separately, for antimatter* in such a way to be locally isomorphic to the spacetime symmetry of electroweak interactions, the latter being an evident condition of consistency.

It is easy to see that, without the prior achievement of a new gravitation possessing an invariance, rather than the covariance of general relativity, any attempt at constructing a grand unification will prove to be futile in due time.

The complexity of the problem is illustrated by the fact that, not only gravitation for matter had to be reformulated in a form admitting a symmetry, but that symmetry had to be compatible with the basic Poincaré symmetry of electroweak theories [4–7]. Moreover, a dual compatible symmetry had to be achieved for the gravity of antimatter.

The latter problems called for a third laborious scientific journey on the *isotopies and isodualities of the Poincaré symmetry* $\hat{P}(3.1)$, today called the *Poincaré-Santilli isosymmetry and its isodual* outlined in Section 3.5 (see monographs [15] for comprehensive studies). These studies included:

1) The isotopies and isodualities of the Lorentz symmetry initiated with paper [18] of 1983 on the classical isotopies with the operator counterpart presented in paper [19] of the same year;

2) The isotopies and isodualities of the rotational symmetry first presented in papers [16][3];

3) The isotopies and isodualities of the $SU(2)$-spin symmetry, first presented in paper [20] of 1993, and related implications for local realist, hidden variables and Bell's inequalities published in Ref. [21] of 1998;

[3]Papers [16] on the lifting of the rotational symmetry were evidently written before paper [19] on the lifting of the Lorentz symmetry, but appeared in print only two years following the latter due to rather unreasonable editorial processing by various journals reported in Ref. [16], which processing perhaps illustrates the conduct of some (but not all) editors when facing true scientific novelty.

4) The isotopies and isodualities of the Poincaré symmetry including the universal invariance of gravitation, first presented in paper [22] of 1993; and

5) The isotopies and isodualities of the spinorial covering of the Poincaré symmetry first presented in papers [23,24] of 1996.[4]

We are referring here to the reconstruction of the conventional symmetries with respect to an arbitrary nonsingular positive-definite unit (14.1.1) for the isotopies, and with respect to an arbitrary nonsingular negative-definite unit (14.1.3) for the isodualities.

This reconstruction yields the most general known nonlinear, nonlocal and noncanonical or nonunitary liftings of conventional symmetries, while the locally isomorphism for isotopies) (anti-isomorphism for isodualities) with the original symmetries is guaranteed by the positive-definiteness (negative-definiteness) of the generalized units.

One should be aware that the above structures required the prior step-by-step isotopies and isodualities of Lie's theory (enveloping associative algebras, Lie algebras, Lie groups, transformation and representation theories, etc.), originally proposed by Santilli in 1978 [12], studied in numerous subsequent works and today called the *Lie-Santilli isotheory and its isodual* (see Section 3.2 for an outline and Refs. [15] for comprehensive studies).

It is evident that the Poincaré-Santilli isosymmetry and its isodual have fundamental character for these studies. One of their primary applications has been the achievement of the universal *symmetry* (rather than covariance) of all possible Riemannian line elements in their iso-Minkowskian representation [22]

$$ds'^2 = dx'^\mu \times g(x')_{\mu\nu} \times dx'^\nu \equiv dx^\mu \times g(x)_{\mu\nu} \times dx^\nu = ds^2, \qquad (14.1.4)$$

[4]Ref. [24], which is the most important reference of this entire monograph (because admitting all topics as particular cases), was rejected for years by all journals of Western Physical Societies because the paper included an *industrial* application currently receiving large investments by the industry — although not by academia, — consisting in the achievement of a numerical, exact and invariant representation of *all* characteristics of the neutron as a bound state of a proton and an electron according to Rutherford. In fact, the resolution of the historical difficulties of Rutherford's conception of the neutron permits the utilization of the large clean energy contained in the neutron's structure, via its stimulated decay caused by a hard photon with a resonating frequency (numerically predicted by hadronic mechanics) that expels Rutherford's electron (the *isoelectron* with an isorenormalized mass generated by the nonlocal and non-Lagrangian interactions in the hyperdense medium inside the proton, see Chapter 6 and references quoted therein),

$$\gamma_{reson.} + n \rightarrow p^+ + e^- + \bar{\nu}.$$

Despite the undeniable mathematical consistency clear plausibility and evident large societal implications due to the need for new clean energies, Ref. [24] was rejected by all Western Physical Society without any credible scientific motivation because not aligned with organized interests in quantum mechanics and special relativity. Paper [24] was finally published in China in 1996. As a gesture of appreciation for this scientific democracy, the author organized in Beijing the 1997 International Workshop on Hadronic Mechanics (see the Proceedings of the Wiorkshops on Hadronic Mechanics listed in the General Bibliography).

Once the unit of gauge theories is lifted to represent gravitation, electroweak interactions will also obey the Poincaré-Santilli isosymmetry for matter and its isodual for antimatter, thus offering realistic hopes for the resolution of the most difficult problem of compatibility between gravitation and electroweak interactions, that for spacetime symmetries.

Perhaps unexpectedly, the fundamental spacetime symmetry of the grand unified theory of Refs. [9–11] is based on the *total symmetry of Dirac's equation*, here written with related spacetime and underlying unit (see Chapter 2 for details)

$$S_{tot} = \{SL(2.C) \times T(3.1) \times \mathcal{I}(1)\} \times \{SL^d(2.C^d) \times^d T^d(3.1) \times^d \mathcal{I}^d(1)\}, \quad (14.1.5a)$$

$$M_{tot} = \{M(x, \eta, R) \times S_{spin}\} \times \{M^d(x^d, \eta^d, R^d) \times^d S^d_{spin}\}, \quad (14.1.5b)$$

$$I_{tot} = \{I_{orb} \times I_{spin}\} \times \{I^d_{orb} \times^d I^d_{spin}\}. \quad (14.1.5c)$$

To understand the above occurrence, the reader should be aware that isodualities imply a new symmetry called *isoselfduality* (Section 2.1), given by the invariance under the isodual map (14.1.2).

Dirac's gamma matrices verify indeed this new symmetry (from which the symmetry itself was derived in the first place), i.e.,

$$\gamma_\mu \rightarrow \gamma^d_\mu = -\gamma^\dagger_\mu = \gamma_\mu. \quad (14.1.6)$$

Consequently, contrary to a popular belief throughout the 20-th century, the Poincaré symmetry *cannot* be the total symmetry of Dirac's equations, evidently because it is not isoselfdual.

For evident reasons of consistency, the total symmetry of Dirac's equation must also be isoselfdual as the gamma matrices are. This condition identifies the total symmetry (14.1.5a) because that symmetry is indeed isoselfdual.

To understand the dimensionality of symmetry (14.1.5a) one must first recall that isodual spaces are independent from conventional spaces. The doubling of the conventionally believed ten-dimensions of the Poincaré symmetry then yields *twenty* dimensions.

But relativistic invariants possess the novel *isotopic invariance* (3.5.27), i.e.,

$$\begin{aligned}(x^\nu \times \eta_{\mu\nu} \times x^\nu) \times I &\equiv [x^\nu \times (w^{-2} \times \eta)_{\mu\nu} \times x^\nu) \times (w^2 \times I) \\ &= (x^\nu \times \hat{\eta}_{\mu\nu} \times x^\nu) \times \hat{I}, \end{aligned} \quad (14.1.7)$$

with corresponding isotopic invariance of Hilbert's inner product

$$\begin{aligned}< \psi| \times |\psi > \times I &\equiv < w^{-1} \times \psi| \times |w^{-1} \times \psi > \times (w^2 \times I) \\ &=< \psi|\hat{\times}|\psi > \times \hat{I}. \end{aligned} \quad (14.1.8)$$

Consequently, the conventional Poincaré symmetry has emerged as being *eleven dimensional* at both the classical and operator levels, as first presented by Santilli in Ref. [22] of 1993 and studied in Section 3.5.3. It then follows that *the total symmetry* (14.1.5a) *of Dirac's equations is twenty-two dimensional.*

The grand unification proposed in Refs. [9–11] is based on the axiomatic structure of the conventional Dirac's equations, not as believed throughout the 20-th century, but as characterized by isotopies and isodualities.

In particular, the grand unification here studied is permitted by the new isotopic invariances (14.1.7) and (14.1.8) that are hidden in relativistic invariants [21], thus assuring the operator compatibility of the grand unification, as we shall see.

The reader should not be surprised that the two new invariances (14.1.7) and (14.1.8) remained undetected throughout the 20-th century because their identification required the prior discovery of *new numbers*, first the numbers with arbitrary positive units, and then the additional new numbers with arbitrary negative units for invariances [25].

(D) Classical and operator isogravitation. After a number of (unpublished) attempts, the resolution of numerous inconsistencies of general relativity studied in Section 1.4, plus the inconsistencies for grand unifications, requested the *isotopic reformulation of gravitation,* today known as *Santilli's isogravitation*, first presented at the VII M. Grossman Meeting on General Relativity of 1996 [26], as reviewed in Section 3.5, essentially consisting in the factorization of any given (nonsingular and symmetric) Riemannian metric $g(x)$ into the Minkowskian metric η multiplied by a 4×4-matrix $\hat{T}$,

$$g(x) = \hat{T}_{Grav}(x) \times \eta, \tag{14.1.9}$$

and the reconstruction of gravitation with respect to the isounit

$$\hat{I}_{Grav}(x) = 1/\hat{T}_{Grav}(x), \tag{14.1.10}$$

thus requiring the isotopic reformulation of the totality of the mathematical and physical methods of general relativity.

Despite its simplicity, the implications of isogravitation are far reaching, such as:

1) The isotopic reformulation permits the achievement of the universal Poincaré-Santilli isoinvariance for all possible gravitational models;

2) The isotopic reformulation eliminates curvature for the characterization of gravity, and replaces it with *isoflatness,* thus achieving compatibility with the flatness of electroweak interactions;

3) The isotopic reformulation reconstructs unitarity on iso-Hilbert spaces over isofields via the identical reformulation of nonunitary transform at the

foundations of hadronic mechanics (Chapter 3)

$$U \times U^\dagger \neq I \to \hat{U} \hat{\times} \hat{U}^\dagger = \hat{U}^\dagger \hat{\times} \hat{U} = \hat{I}_{Grav}, \tag{14.1.11}$$

where

$$U \times U^\dagger = \hat{I}, \quad \hat{U} = U \times \hat{T}_{Grav}^{1/2}, \tag{14.1.12}$$

thus providing the *only* known resolution of the catastrophic inconsistencies of Theorems 1.5.1 and 1.5.2.

Above all, isogravitation achieved the first and only known, axiomatically consistent operator formulation of gravitation provided by relativistic hadronic mechanics of Section 3.5, as first presented in Ref. [27] of 1997.

In fact, gravity is merely imbedded in the *unit* of relativistic operator theories. Since the gravitational isounit is positive-definite from the nonsingular and symmetric character of the metric $g(x)$ in factorization (14.1.9), the abstract axioms of operator isogravity are the conventional axioms of relativistic quantum mechanics, only subjected to a broader realization.

The preservation of conventional relativistic axioms then assures the achievement, for the first time as known by the author, of a consistent operator formulation of gravitation.[5]

(E) Geometric unification of special and general relativities. The resolution of the problems caused by lack of any Minkowskian limit of general relativity requested additional studies. After a number of (unpublished) attempts, the only possible solution resulted to be a geometric unification of special and general relativities, first presented in Ref. [28], in which the two relativities are characterized

by the same abstract axioms and are differentiated only by their realization of the basic unit. The trivial realization $I = Diag.(1,1,1,1)$ characterizes special relativity, and broader realization (14.1.10) characterizes general relativity.

The latter final efforts requested the construction *ab initio* of a new geometry, today known as *Minkowski-Santilli isogeometry* [28] in which the abstract axioms are those of the Minkowskian geometry, including the abstract axiom of flatness necessary to resolve the catastrophic inconsistencies of Section 1.4,

[5] Note that the use of the words "quantum gravity" for operator formulation of gravitation, whether conventional or characterized by the isotopies, would be merely political. This is due to the fact that, on serious scientific grounds, the term "quantum" can only be referred to physical conditions admitting a quantized emission and absorption of energy as occurring in the structure of the hydrogen atom. By comparison, no such quantized orbits are possible for operator theories of gravity, thus rendering nonscientific its characterization as "quantum gravity". Ironically, the editor of a distinguished physics journal expressed interest in publishing a paper on "operator isogravity" under the condition of being called "quantum gravity", resulting in the necessary withdrawal of the paper by the author so as not to reduce fundamental physical inquiries to political compromises.

yet the new geometry admits the entire mathematical formalist of the Riemannian geometry, including covariant derivatives, Christoffel's symbols, etc. (see Section 3.2 for an outline and monographs [15] for comprehensive studies).

The important point is that at the limit

$$\lim\ \hat{I}_{Grav}(x) \rightarrow I, \tag{14.1.13}$$

the Minkowskian geometry and conventional special relativity are recovered identically and uniquely.

The reader should be aware that the grand unification presented in this section is centrally dependent on the Minkowski-Santilli isogeometry, the Poincaré-Santilli isosymmetry, and the isotopic formulation of gravitation. Their knowledge is a necessary pre-requisite for the technical understanding of the following sections.

14.1.4 Isotopic Gauge Theories

The isotopies of gauge theories were first studied in the 1980's by Gasperini [29], followed by Nishioka [30], Karajannis and Jannussis [31] and others, and ignored thereafter for over a decade.

These studies were defined on conventional spaces over conventional fields and were expressed via the conventional differential calculus. As such, they are not invariant, as it became shown in memoirs [32], thus suffering of the catastrophic inconsistencies of Theorem 1.5.2.

Refs. [9–11] presented, apparently for the first time, the *invariant isotopies of gauge theories*, or *isogauge theories* for short, and their isoduals, those formulated on isospaces over isofields and characterized by the isodifferential calculus of memoir [14]. For completeness, let us recall that the latter theories are characterized by the following methods:

(1) Isofields [25] of isoreal numbers $\hat{R}(\hat{n}, \hat{+}, \hat{\times})$ and isocomplex numbers $\hat{C}(\hat{c}, \hat{+}, \hat{\times})$ with: additive isounit $\hat{0} = 0$; generalized multiplicative isounit $\hat{I}$ given by Eq. (14.1.9); elements, isosum, isoproduct and related generalized operations,

$$\hat{a} = a \times \hat{I},\ \ \hat{a}\hat{+}\hat{b} = (a + b) \times \hat{I}, \tag{14.1.14a}$$

$$\hat{a}\hat{\times}\hat{b} = \hat{a} \times \hat{T} \times \hat{b} = (a \times b) \times \hat{I}, \tag{14.1.14b}$$

$$\hat{a}^{\hat{n}} = \hat{a}\hat{\times}\hat{a}\hat{\times}...\hat{\times}\hat{a}, \tag{14.1.14c}$$

$$\hat{a}^{\hat{1/2}} = a^{1/2} \times \hat{I}^{1/2},\ \ \hat{a}\hat{/}\hat{b} = (\hat{a}/\hat{b}) \times \hat{I},\ \text{etc.} \tag{14.1.14d}$$

(2) Isominkowski spaces [18] $\hat{M} = \hat{M}(\hat{x}, \hat{\eta}, \hat{R})$ with isocoordinates $\hat{x} = x \times \hat{I} = \{x^\mu\} \times \hat{I}$, isometric $\hat{N} = \hat{\eta} \times \hat{I} = [\hat{T}(x, \dots) \times \eta] \times \hat{I}$, and *isointerval* over the isoreals $\hat{R}$

$$(\hat{x} - \hat{y})^{\hat{2}} = [(\hat{x} - \hat{y})^\mu \hat{\times} \hat{N}_{\mu\nu} \hat{\times} (\hat{x} - \hat{y})^\nu]$$

$$= [(x - y)^{\mu} \times \hat{\eta}_{\mu\nu} \times (x - y)^{\nu}] \times \hat{I}, \tag{14.1.15}$$

equipped with *Kadeisvili isocontinuity* [33] and the *isotopology* developed by G. T. Tsagas and D. S. Sourlas [34], R. M. Santilli [14], R. M. Falcón Ganfornina and J. Núñez Valdés [35,36] (see also Aslander and Keles [37]). A more technical formulation of the isogauge theory can be done via the isobundle formalism on isogeometries.

(3) Isodifferential calculus [14] characterized by the following isodifferentials

$$\hat{d}\hat{x}^{\mu} = \hat{I}^{\mu}_{\nu} \times d\hat{x}^{\nu}, \tag{14.1.16a}$$

$$\hat{d}\hat{x}_{\mu} = \hat{T}^{\nu}_{\mu} \times d\hat{x}_{\nu}, \tag{14.1.16b}$$

and isoderivatives

$$\hat{\partial}_{\mu}\hat{f} = \hat{\partial}\hat{f}\hat{/}\hat{\partial}\hat{x}^{\mu} = (\hat{T}^{\nu}_{\mu} \times \partial_{\nu} f) \times \hat{I}, \tag{14.1.17a}$$

$$\hat{\partial}^{\mu}\hat{f} = (\hat{I}^{\mu}_{\nu} \times \partial_{\nu} f) \times \hat{I}, \;\; \hat{\partial}\hat{x}^{\mu}\hat{/}\hat{\partial}\hat{x}^{\nu} = \hat{\delta}^{\mu}_{\nu} = \delta^{\mu}_{\nu} \times \hat{I}, \text{ etc.} \tag{14.1.17b}$$

where one should note the inverted use of the isounit and isotopic element with respect to preceding formulations.

(4) Isofunctional isoanalysis [15], including the reconstruction of all conventional and special functions and transforms into a form admitting of $\hat{I}_{Grav}$ as the left and right unit. Since the iso-Minkowskian geometry preserves the Minkowskian axioms, it allows the preservation of the notions of straight and intersecting lines, thus permitting the reconstruction of trigonometric and hyperbolic functions for the Riemannian metric $g(x) = \hat{T}(x) \times \eta$.

(5) Iso-Minkowskian geometry [28], i.e., the geometry of isomanifolds $\hat{M}$ over the isoreals $\hat{R}$, that satisfies all abstract Minkowskian axioms because of the joint liftings

$$\eta \rightarrow \hat{\eta} = T(x, \dots) \times \eta, \tag{14.1.18a}$$

$$I \rightarrow \hat{I} = T^{-1}, \tag{14.1.18b}$$

while preserving the machinery of Riemannian spaces as indicated earlier, although expressed in terms of the isodifferential calculus.

In this new geometry *Riemannian* line elements are turned into identical *Minkowskian* forms via the embedding of gravity in the deferentials, e.g., for the Schwarzschild exterior metric we have the iso-Minkowskian reformulation (Ref. [28], Eqs. (2.57)), where the spacetime coordinates are assumed to be covariant,

$$\hat{d}\hat{s} = \hat{d}\hat{r}^{\hat{2}} \hat{+} \hat{r}^{\hat{2}} \hat{\times} (\hat{d}\hat{\theta}^{\hat{2}} \hat{+} isosin^{\hat{2}}\hat{\theta}) \hat{-} \hat{d}\hat{t}^{\hat{2}}, \tag{14.1.19a}$$

$$\hat{d}\hat{r} = \hat{T}_r \times d\hat{r}, \hat{d}\hat{t} = \hat{T}_t \times d\hat{t}, \tag{14.1.19b}$$

$$\hat{T}_r = (1 - 2 \times M/r)^{-1}, \;\; \hat{T}_t = 1 - 2 \times M/r. \tag{14.1.19c}$$

(6) Relativistic hadronic mechanics [15] characterized by the *iso-Hilbert space* $\hat{\mathcal{H}}$ with *isoinner product and isonormalization* over $\hat{C}$

$$< \hat{\phi}|\hat{\times}|\hat{\psi} > \times \hat{I}, \quad < \hat{\psi}|\hat{\times}|\hat{\psi} >= \hat{I}. \tag{14.1.20}$$

Among various properties, we recall that: the *iso-Hermiticity* on $\hat{\mathcal{H}}$ coincides with the conventional Hermiticity (thus, all conventional observables remain observables under isotopies); the isoeigenvalues of iso-Hermitean operators are real and conventional (because of the identities

$$\hat{H}\hat{\times}|\hat{\psi} >= \hat{E}\hat{\times}|\hat{\psi} >= E \times |\hat{\psi} >; \tag{14.1.21}$$

the condition of *isounitarity* on $\hat{\mathcal{H}}$, over $\hat{C}$ is given by

$$\hat{U}\hat{\times}\hat{U}^{\dagger} = \hat{U}^{\dagger}\hat{\times}\hat{U} = \hat{I}, \tag{14.1.22}$$

(see memoir [27] for details).

(7) The Lie-Santilli isotheory [12] with: conventional (ordered) basis of generators $X = (X_k)$, and parameters $w = (w_k)$, $k = 1, 2, \ldots, n$, only formulated in isospaces over isofields with a common isounit; universal enveloping isoassociative algebras $\hat{\xi}$ with infinite-dimensional basis characterized by the isotopic Poincare'-Birkhoff-Witt theorem [12]

$$\hat{I}, \ \hat{X}_i\hat{\times}\hat{X}_j, \ (i \leq j), \quad \hat{X}_i\hat{\times}\hat{X}_j \times \hat{X}_k, \ (i \leq j \leq k, ...) \tag{14.1.23}$$

Lie-Santilli subalgebras [12]

$$[\hat{X}_i \hat{,} \hat{X}_j] = \hat{X}_i\hat{\times}\hat{X}_j - \hat{X}_j\hat{\times}\hat{X}_i = \hat{C}^k_{ij}(x, \ldots)\hat{\times}\hat{X}_k, \tag{14.1.24}$$

where the $\hat{C}$'s are the structure disfunctions; and isogroups characterized by isoexponentiation on $\hat{\xi}$ with structure [12]

$$\hat{e}^{\hat{X}} = \hat{I}\hat{+}\hat{X}\hat{/}\hat{1}\hat{!}\hat{+}\hat{X}\hat{\times}\hat{X}\hat{/}\hat{2}\hat{!}\hat{+}... = (e^{\hat{X}\times\hat{T}}) \times \hat{I} = \hat{I} \times (e^{\hat{T}\times\hat{X}}). \tag{14.1.25}$$

Despite the isomorphism between isotopic and conventional structures, the lifting of Lie's theory is nontrivial because of the appearance of the matrix $\hat{T}$ with nonlinear integrodifferential elements in the very *exponent* of the group structure, Eqs. (14.1.25).

To avoid misrepresentations, one should keep in mind that the isotopies of Lie's theory *were not* proposed to identify "new Lie algebras" (an impossible task since all simple Lie algebras are known from Cartan's classification), but to construct instead the most general possible nonlinear, nonlocal and noncanonical or nonunitary "realizations" of known Lie algebras.

(8) Isolinearity, isolocality and isocanonicity or isounitarity. Recall from lifting (14.1.25) that isosymmetries have the most general possible nonlinear, nonlocal and noncanonical or nonunitary structure. A main function of the isotopies is that of reconstructing linearity, locality and canonicity or unitarity on isospaces over isofields, properties called *isolinearity, isolocality and isocanonicity or isounitarity.* These are the properties that permit the bypassing of the theorems of catastrophic inconsistencies of Section 1.5.

As a result, the use of the conventional *linear* transformations on M over R, $X' = A(w) \times x$ violates *isolinearity* on $\hat{M}$ over $\hat{R}$.

In general, *any* use of conventional mathematics for isotopic theories leads to a number of inconsistencies which generally remain undetected by nonexperts in the field.[6]

(9) Isogauge theories [9–11]. They are characterized by an n-dimensional connected and non-isoabelian isosymmetry $\hat{G}$ with: basic n-dimensional isounit (4.1.9); iso-Hermitean generators $\hat{X}$ on an iso-Hilbert space $\hat{\mathcal{H}}$ over the isofield $\hat{C}(\hat{c}, \hat{+}, \hat{\times})$; universal enveloping associative algebra $\hat{\xi}$ with infinite isobasis (14.1.23); isocommutation rules (14.1.24); isogroup structure

$$\hat{U} = \hat{e}^{-i\times X_k\times\theta(x)_k} = (e^{-i\times X_k\times\hat{T}\times\theta(x)_k}) \times \hat{I}, \quad \hat{U}^{\dagger}\hat{\times}\hat{U} = \hat{I}, \tag{14.1.26}$$

where one should note the appearance of the gravitational isotopic elements in the *exponent* of the isogroup, and the parameters $\theta(x)_k$ now depend on the iso-Minkowski space; isotransforms of the isostates on $\hat{\mathcal{H}}$

$$\hat{\psi}' = \hat{U}\hat{\times}\hat{\psi} = \left(e^{-i\times X_k\times\hat{T}(x,\dots)\times\theta(x)_k}\right) \times \hat{\psi}; \tag{14.1.27}$$

isocovariant derivatives [28]

$$\hat{D}_\mu\hat{\psi} = (\hat{\partial}_\mu - i\hat{\times}\hat{g}\hat{\times}\hat{A}(\hat{x})^k_\mu\hat{\times}\hat{X}_k)\hat{\times}\hat{\psi}; \tag{14.1.28}$$

iso-Jacobi identity

$$[\hat{D}_\alpha\hat{,}[\hat{D}_\beta\hat{,}\hat{D}_\gamma]] \hat{+} [\hat{D}_\beta\hat{,}[\hat{D}_\gamma\hat{,}\hat{D}_\alpha]] \hat{+} [\hat{D}_\gamma\hat{,}[\hat{D}_\alpha\hat{,}\hat{D}_\beta]] = 0, \tag{14.1.29}$$

where g and $\hat{g} = g \times \hat{I}$ are the conventional and isotopic coupling constants, $A(x)^k_\mu \times X_k$ and $\hat{A}(\hat{x})^k_\mu\hat{\times}\hat{X}_k = [A(x)^k_\mu \times X_k] \times \hat{I}$ are the gauge and isogauge potentials; isocovariance

$$(\hat{D}_\mu\hat{\psi})' = (\hat{\partial}_\mu\hat{U})\hat{\times}\hat{\psi}\hat{+}\hat{U}\hat{\times}(\hat{\partial}_\mu\hat{\psi})\hat{-}\hat{i}\hat{\times}\hat{g}\hat{\times}\hat{A}'(\hat{x})_\mu\hat{\times}\hat{\psi} = \hat{U}\hat{\times}\hat{D}_\mu\hat{\psi}, \tag{14.1.30a}$$

$$\hat{A}(\hat{x})'_\mu = -\hat{g}^{-\hat{1}}\hat{\times}[\hat{\partial}_\mu\hat{U}(\hat{x})]\hat{\times}\hat{U}(\hat{x})^{-\hat{1}}, \tag{14.1.30b}$$

$$\hat{\delta}\hat{A}(\hat{x})^k_\mu = -\hat{g}^{-\hat{1}} \hat{\times} \hat{\partial}_\mu\hat{\theta}(\hat{x})^k \hat{+} \hat{C}^k_{ij} \hat{\times} \hat{\theta}(\hat{x})^i \hat{\times} \hat{A}(\hat{x})^j_\mu, \tag{14.1.30c}$$

[6] The use of conventional mathematics for isotheories would be the same as elaborating Balmer's quantum spectral lines in the hydrogen atoms with isofunctional analysis, resulting in evident major inconsistencies.

$$\hat{\delta}\hat{\psi} = -\hat{i}\hat{\times}\hat{g}\hat{\times}\hat{\theta}(\hat{x})^k\hat{\times}\hat{X}_k\hat{\times}\hat{\psi}; \tag{14.1.30d}$$

non-isoabelian iso-Yang-Mills fields

$$\hat{F}_{\mu\nu} = \hat{i}\hat{\times}\hat{g}^{-\hat{1}}\hat{\times}[\hat{D}_{\hat{\mu}}, \hat{D}_\nu]\hat{\psi}, \tag{14.1.31a}$$

$$\hat{F}^k_{\mu\nu} = \hat{\partial}_\mu\hat{A}^k_\nu \hat{-} \hat{\partial}_\nu\hat{A}^k_\mu \hat{+} \hat{g}\hat{\times}\hat{C}^k_{ij}\hat{\times}\hat{A}^i_\mu\hat{\times}\hat{A}^j_\nu; \tag{14.1.31b}$$

related isocovariance properties

$$\hat{F}_{\mu\nu} \rightarrow \hat{F}'_{\mu\nu} = \hat{U}\hat{\times}\hat{F}_{\mu\nu}\hat{\times}\hat{U}^{-1}, \tag{14.1.32a}$$

$$Isotrace(\hat{F}_{\mu\nu'}\hat{\times}\hat{F}^{\mu\nu'}) = Isotrace(\hat{F}_{\mu\nu}\hat{\times}\hat{F}^{\mu\nu}), \tag{14.1.32b}$$

$$[\hat{D}_\alpha\hat{,}\hat{F}_{\beta\gamma}] \hat{+} [\hat{D}_\beta\hat{,}\hat{F}_{\gamma\alpha}] \hat{+} [\hat{D}_\gamma\hat{,}\hat{F}_{\alpha`\beta}] \equiv 0; \tag{14.1.32c}$$

derivability from the isoaction

$$\hat{S} = \hat{\int} \hat{d}^{\hat{4}}\hat{x}(-\hat{F}_{\mu\nu}\hat{\times}\hat{F}^{\mu\nu}\hat{/}\hat{4}) = \hat{\int} \hat{d}^{\hat{4}}\hat{x}(-\hat{F}^k_{\mu\nu}\hat{\times}\hat{F}^{\mu\nu}_k\hat{/}\hat{4}), \tag{14.1.33}$$

where $\hat{\int} = \int \times \hat{I}$, plus all other familiar properties in isotopic formulation.

The *isodual isogauge theory*, first proposed in Refs. [9–11], is the image of the preceding theory under the isodual map (14.1.2) when applied to the totality of quantities and their operations.

The latter theory is characterized by the isodual isogroup $\hat{G}^d$ with isodual isounit

$$\hat{I}^d_{Grav} = -\hat{I}^\dagger_{Grav} = -\hat{I}_{Grav} = -1/\hat{T}_{Grav} < 0. \tag{14.1.34}$$

The elements of the base fields

$$\hat{R}^d(\hat{n}^d, \hat{+}^d, \hat{\times}^d), \tag{14.1.35}$$

are given by the isodual isoreal numbers

$$\hat{n}^d = -\hat{n} = -n \times \hat{I}, \tag{14.1.36}$$

and those of the field

$$\hat{C}^d(\hat{c}^d, \hat{+}^d, \hat{\times}^d), \tag{14.1.37}$$

are the isodual isocomplex numbers

$$\hat{c}^d = -(c \times \hat{I})^\dagger = (n_1 - i \times n_2) \times \hat{I}^d = (-n_1 + i \times n_2) \times \hat{I}. \tag{14.1.38}$$

The carrier spaces are the isodual iso-Minkowski spaces $\hat{M}^d(\hat{x}^d, -\hat{\eta}^d, \hat{R}^d)$ on $\hat{R}^d$ and the isodual iso-Hilbert space $\mathcal{H}^d$ on $\hat{C}^d$ with isodual isostates and isodual isoinner product

$$|\hat{\psi}>^d = -|\hat{\psi}>^\dagger = -<\psi|, \tag{14.1.39a}$$

$$< \hat{\phi}|^d \times \hat{T}^d \times |\hat{\psi} >^d \times \hat{I}^d. \qquad (14.1.39b)$$

It is instructive to verify that all eigenvalues of isodual iso-Hermitean operators are *negative − definite* (when projected in our space-time),

$$\hat{H}^d \hat{\times}^d |\hat{\psi} >^d = < \psi| \times (-E). \qquad (14.1.40)$$

$\hat{G}^d$ is characterized by the isodual Lie-Santilli isotheory with isodual generators $\hat{X}^d = -\hat{X}$, isodual isoassociative product

$$\hat{A}^d \hat{\times}^d \hat{B}^d = \hat{A}^d \times \hat{T}^d \times \hat{B}^d, \;\; \hat{T}^d = -\hat{T}, \qquad (14.1.41)$$

and related isodual isoenveloping and Lie-Santilli isoalgebra.

The elements of $\hat{G}^d$ are the isodual isounitary isooperators

$$\hat{U}^d(\hat{\theta}^d(\hat{x}^d)) = -\hat{U}^\dagger(-\hat{\theta}(-\hat{x})). \qquad (14.1.42)$$

In this way, the isodual isogauge theory is seen to be an anti-isomorphic image of the preceding theory, as desired.

It is an instructive exercise for the reader interested in learning the new techniques to study first the isodualities of the *conventional* gauge theory (rather than of their isotopies), and show that they essentially provide a mere reinterpretation of the usually discarded, advanced solutions as characterizing antiparticles.

Therefore, in the isoselfdual theory with total gauge symmetry $\hat{G} \times \hat{G}^d$, isotopic retarded solutions are associated with particles and advanced isodual solutions are associated with antiparticles.

No numerical difference is expected in the above reformulation because, as shown in Chapter 3, isotopies preserve not only the original axioms but also the original numerical value (when constructed properly).

It is also recommendable for the interested reader to verify that the isotopies are indeed equivalent to charge conjugation for all massive particles, with the exception of the photon (see Section 2.3). In fact, isodual theories predict that the antihydrogen atom emits a new photon, tentatively called by this author the *isodual photon* [38], that coincides with the conventional photon for all possible interactions, thus including electroweak interactions, *except gravitation.* This indicates that the isodual map is inclusive of charge conjugation for massive particles, but it is broader than the latter.

Isodual theories in general, thus including the proposed grand unification, predict that all *stable* isodual particles, such as the isodual photon, the isodual electron (positron), the isodual proton (antiproton) and their bound states (such as the antihydrogen atom), experience *antigravity* in the field of the Earth (defined as the reversal of the sign of the curvature tensor).

If confirmed, the prediction may offer the possibility in the future to ascertain whether far away galaxies and quasars are made-up of matter or of antimatter.

We finally note that isomathematics is a particular case of the broader *genomathematics*, also introduced for the first time in Refs. [12] of 1978 (see Chapter 4), which occurs for non-Hermitean generalized units and is used for an axiomatization of irreversibility.

In turn, genomathematics is a particular case of the *hypermathematics*, that occurs when the generalized units are given by ordered *sets* of non-Hermitean quantities and is used for the representation of multivalued complex systems (e.g. biological entities) in irreversible conditions.

Evidently both the genomathematics and hypermathematics admit an anti-isomorphic image under isoduality (see also Chapter 4).

In conclusion the methods outlined in this note permit the study of *seven* liftings of conventional gauge theories [9–11]:

(1) The *isodual gauge theories* for the treatment of antimatter without gravitation in vacuum;

(2,3) The *isogauge theories and their isoduals*, for the inclusion of gravity for matter and antimatter in reversible conditions in vacuum (exterior gravitational problem);

(4,5) The *genogauge theories and their isoduals*, for the inclusion of gravity for matter and antimatter in irreversible interior conditions (interior gravitational problems); and

(6,7) the *hypergauge theories and their isoduals*, for multivalued and irreversible generalizations.

For brevity this section is restricted to theories of type (1), (2), (3). The development of the remaining genotopies of gauge theories is left to interested readers.

14.1.5 Iso-, Geno- and Hyper-Grand-Unifications

In this section we review the *Iso-Grand-Unification* (IGU) with the inclusion of electroweak and gravitational interactions, first submitted in Refs. [9–11] via the 22-dimensional total isoselfdual isosymmetry given by isosymmetry (3.5.28) and its isodual

$$\hat{S}_{tot} = (\hat{\mathcal{P}}(3.1)\hat{\times}\hat{G}) \times (\hat{\mathcal{P}}(3.1)^d\hat{\times}^d\hat{G}^d) =$$

$$= [\hat{SL}(2,\hat{C})\hat{\times}\hat{T}(3.1)\hat{\times}\hat{\mathcal{I}}(1)] \times [\hat{SL}^d(2,\hat{C}^d)\hat{\times}^d\hat{T}^d(3.1)\hat{\times}^d\hat{\mathcal{I}}^d(1)], \tag{14.1.43}$$

where $\hat{\mathcal{P}}$ is the Poincaré-Santilli isosymmetry [22] in its isospinorial realization [24], $\hat{G}$ is the isogauge symmetry of the preceding section and the remaining structures are the corresponding isoduals.

Without any claim of a final solution, it appears that the proposed IGU does indeed offer realistic possibilities of resolving the axiomatic incompatibilities (1)–(5) of Section 14.1.2 between gravitational and electroweak interactions.

In fact, IGU represents gravitation in a form geometrically compatible with that of the electroweak interactions, represents antimatter at all levels via negative-energy solutions, and characterizes both gravitation as well as electroweak interactions via the universal Poincaré-Santilli isosymmetry.

It should be indicated that we are referring here to the *axiomatic* consistency of IGU. In regard to the *physical* consistency we recall that isotopic liftings preserve not only the original axioms, but also the original numerical values [15].

As an example, the image in iso-Minkowskian space over the isoreals of the light cone, the isolight cone, not only is a perfect cone, but a cone with the original characteristic angle, thus preserving the speed of light in vacuum as the maximal causal speed in iso-Minkowskian space.

This peculiar property of the isotopies implies the expectation that the proposed Iso-Grand-Unification preserves the numerical results of electroweak interactions.

The reader should be aware that the methods of the recent memoir [27] permit a truly elementary, explicit construction of the proposed IGU.

As well known, the transition from the Minkowskian metric η to Riemannian metrics $g(x)$ is a *noncanonical transform* at the classical level, and, therefore, a at the operator level.

The method herein considered for turning a gauge theory into an IGU consists in the following representation of the selected gravitational model, e.g., Schwarzschild's model:

$$g(x) = T(x) \times \eta, \tag{14.1.44a}$$

$$I(x) = U \times U^\dagger = 1/\hat{T} =$$

$$Diag.[(1 - 2 \times M/r) \times Diag.(1,1,1),\ (1 - 2 \times M/r)^{-1}], \tag{14.1.44b}$$

and then subjecting the *totality* of the gauge theory to the nonunitary transform $U \times U^\dagger$.

The method then yields: the isounit

$$I \to \hat{I} = U \times I \times U^\dagger; \tag{14.1.45}$$

the isonumbers

$$a \to \hat{a} = U \times a \times U^\dagger = a \times (U \times U^\dagger) = a \times \hat{I},\ a = n, c; \tag{14.1.46}$$

the isoproduct with the correct expression and Hermiticity of the isotopic element,

$$A \times B \to U \times (A \times B) \times U^\dagger =$$

$$= (U \times A \times U^\dagger) \times (U \times U^\dagger)^{-1} \times (U \times B \times U^\dagger) =$$
$$= \hat{A} \times \hat{T} \times \hat{B} = \hat{A} \hat{\times} \hat{B}; \tag{14.1.47}$$

the correct form of the iso-Hilbert product on $\hat{C}$,

$$< \phi| \times |\psi > \rightarrow U \times < \phi| \times |\psi > \times U^\dagger =$$
$$= (< \psi| \times U^\dagger) \times (U \times U^\dagger)^{-1} \times (U \times |\psi >) \times (U \times U^\dagger) =$$
$$=< \hat{\phi}| \times \hat{T} \times |\hat{\psi} > \times \hat{I}; \tag{14.1.48}$$

the correct Lie-Santilli isoalgebra

$$A \times B - B \times A \rightarrow \hat{A} \hat{\times} \hat{B} - \hat{B} \hat{\times} \hat{A}; \tag{14.1.49}$$

the correct isogroup

$$U \times (e^X) \times U^\dagger = (e^{X \times \hat{T}}) \times \hat{I}, \tag{14.1.50}$$

the Poincaré-Santilli isosymmetry $\mathcal{P} \rightarrow \hat{\mathcal{P}}$, and the isogauge group $G \rightarrow \hat{G}$.

It is then easy to verify that the emerging IGU is indeed invariant under all possible additional nonunitary transforms, provided that, for evident reasons of consistency, they are written in their identical isounitary form,

$$W \times W^\dagger = \hat{I}, \tag{14.1.51a}$$

$$W = \hat{W} \times \hat{T}^{1/2}, W \times W^\dagger = \hat{W} \hat{\times} \hat{W}^\dagger = \hat{W}^\dagger \hat{\times} \hat{W} = \hat{I}. \tag{14.1.51b}$$

In fact, we have the invariance of the isounit

$$\hat{I} \rightarrow \hat{I}' = \hat{W} \hat{\times} \hat{I} \hat{\times} \hat{W}^\dagger = \hat{I}, \tag{14.1.52}$$

the invariance of the isoproduct

$$\hat{A} \hat{\times} \hat{B} \rightarrow \hat{W} \hat{\times} (\hat{A} \hat{\times} \hat{B}) \hat{\times} \hat{W}^\dagger = \hat{A}' \hat{\times} \hat{B}', \text{ etc.} \tag{14.1.53}$$

Note that the isounit is *numerically* preserved under isounitary transforms, as it is the case for the conventional unit I under unitary transform, and that the selection of a nonunitary transform $W \times W^\dagger = \hat{I}'$ with value different from $\hat{I}$ evidently implies the transition to a different gravitational model.

Note that the lack of implementation of the above nonunitary-isounitary lifting to only *one* aspect of the original gauge theory (e.g., the preservation of the old numbers or of the old differential calculus) implies the loss of the invariance of the theory [32].

The assumption of the negative-definite isounit $\hat{I}^d = -(U \times U^\dagger)$ then yields the isodual component of the IGU.

Note finally that diagonal realization (14.1.44) has been assumed mainly for simplicity. In general, the isounit is positive-definite but *nondiagonal* 4×4-dimensional matrix. The Schwarzschild metric can then be more effectively represented in its isotropic coordinates as studied, e.g. in Ref. [39], pp. 196–199).

In closing, the most significant meaning of IGU is that *gravitation has always been present in unified gauge theories. It did creep-in un-noticed because embedded where nobody looked for, in the "unit" of gauge theories.*

In fact, the isogauge theory of Section 14.1.4 coincides with the conventional theory at the abstract level to such an extent that we could have presented IGU with exactly the same symbols of the conventional gauge theories without the "hats", and merely subjecting the same symbols to a more general realization.

Also, the isounit representing gravitation as per rule (14.1.9) verifies all the properties of the conventional unit I of gauge theories,

$$\hat{I}^{\hat{n}} = \hat{I}, \quad \hat{I}^{\hat{1/2}} = \hat{I}, \tag{14.1.54a}$$

$$d\hat{I}/dt = \hat{I}\hat{\times}\hat{H} - \hat{H}\hat{\times}\hat{I} = \hat{H} - \hat{H} = 0, \text{ etc.} \tag{14.1.54b}$$

The "hidden" character of gravitation in conventional gauge theories is then confirmed by the isoexpectation value of the isounit recovering the conventional unit I of gauge theories,

$$\hat{<}\hat{I}\hat{>} = < \hat{\psi}| \times \hat{T} \times \hat{I} \times \hat{T} \times |\hat{\psi} > / < \hat{\psi}| \times \hat{T} \times |\hat{\psi} >= I. \tag{14.1.55}$$

It then follows that *IGU constitutes an explicit and concrete realization of the theory of "hidden variables"* [40]

$$\lambda = T(x) = g(x)/\eta, \hat{H}\hat{\times}|\hat{\psi} >= \hat{H} \times \lambda \times |\hat{\psi} >= E_\lambda \times |\hat{\psi} >, \tag{14.1.56}$$

and the theory is correctly reconstructed with respect to the new unit

$$\hat{I} = \lambda^{-1}, \tag{14.1.57}$$

in which von Neumann's Theorem [41] and Bell's inequalities [42] do not apply, evidently because of the nonunitary character of the theory (see Ref. [21] and Vol. II of Refs. [15] for details).

In summary, the proposed inclusion of gravitation in unified gauge theories is essentially along the teaching of Einstein, Podolsky, and Rosen [43] on the "lack of completion" of quantum mechanics, only applied to gauge theories.

14.2 ISO-, GENO-, AND HYPER-SELF-DUAL COSMOLOGIES

A rather popular belief of the 20-th century was that the universe is solely composed of matter. This belief was primarily due to the scientific imbalance pertaining to antimatter as being solely studied at the level of second quantization, without any theoretical, let alone experimental, mean available for the study of antimatter.

In reality, there exists rather strong evidence that the universe is indeed composed of matter as well as antimatter and, more particularly, that some of the galaxies are made up of matter and others of antimatter.

To begin, not only the expansion of the universe, but more particularly the recently detected increase of the expansion itself, can be readily explained via an equal distribution of matter and antimatter galaxies.

In fact, antigravity experienced by matter and antimatter galaxies (studied in the preceding chapter) explains the expansion of the universe, while the continuous presence of antigravity explains the increase of the expansion.

The assumption that the universe originated from a primordial explosion, the "big bang", could have explained at least conceptually the expansion of the universe. However, the "big bang" conjecture is eliminated as scientifically possible by the increase of the expansion itself.

The "big bang" conjecture is also eliminated by the inability to explain a possible large presence of antimatter in the universe, trivially, because it would have been annihilated at the time of the "big bang" because produced jointly with matter, as well as for other reasons.

By comparison, the only plausible interpretation at the current state of our knowledge is precisely the assumption that the universe is made up half of *matter galaxies* and half of *antimatter galaxies* due to the joint explanation of the expansion of the universe and its increase.

Independently from the above, there exists significant evidence that our Earth is indeed bombarded by antimatter particles and asteroids.

Astronauts orbiting Earth in spaceship have systematically reported that, when passing over the dark side, they see numerous flashes in the upper atmosphere that can be only interpreted as *antimatter cosmic rays*, primarily given by high energy antiprotons and/or positrons[7] originating from far away antimatter galaxies, which antiparticles, when in contact with the upper layers of our atmosphere, annihilate themselves producing the flashes seen by astronauts.

Note that the conventional *cosmic rays* detected in our atmosphere are *matter cosmic rays,* that is, high energy *particles*, such as protons and electrons, originating from a matter supernova or other matter astrophysical event.

In any case, it is evident that matter cosmic rays with sufficient energy can indeed penetrate deep into our atmosphere, while antimatter cosmic rays will be stopped by the upper layers of our atmosphere irrespective of their energy.

[7] Evidently only *stable antiparticles can travel intergalactic distances without decaying.*

In addition, there exists evidence that our Earth has been hit by *antimatter meteorites* that, as such, can only originate from an astrophysical body made up of antimatter.

The best case is the very large devastation re corded in 1908 in Tunguska, Siberia, in which over one million acres of forest were completely flattened in a radial direction originating from a common center without any crater whatever, not even at the center.

The lack of a crater combined with the dimension of the devastation, exclude the origination from the explosion of a *matter asteroid*, firstly, because in this case debris would have been detected by the various expeditions in the area and, secondly, because there is no credible possibility that the mere explosion of a matter asteroid could have caused a devastation over such a large area requiring energies computed at about 100 times the atomic bomb exploded over Hiroshima, Japan.

The only plausible interpretation of the *Tunguska explosion* is that it was due to an antimatter asteroid that eventually annihilated after contact deep into our matter atmosphere.

The important point is that the numerical understanding of the Tunguska explosion requires an antimatter mass of the order of a ton, namely, an antimatter asteroid that, as such, can only originate from the supernova explosion of an antimatter star.

Consequently, the evidence on the existence of even one antimatter asteroid confirms the existence in the universe of antimatter stars. Since it is highly improbable that antimatter stars can exist within a matter galaxy, antimatter asteroids constitute significant evidence on the existence in the universe of antimatter galaxies.

But again, the expansion of the universe as well as the increase of the expansion itself are the strongest evidence for an essentially equal distribution of matter and antimatter galaxies in the universe, as well as for the existence of antigravity between matter and antimatter.

In any case, there exist no alternative hypothesis at all known to this author, let alone a credible hypothesis, that could explain quantitatively both the expansion of the universe and the increase of the expansion itself.

In view of the above occurrences, as well as to avoid discontinuities at creation, Santilli [44] proposed the new *Iso-Self-Dual Cosmology*, namely, a cosmology in which the universe has an exactly equal amount of matter and antimatter, much along the isoselfdual re-interpretation of Dirac's equation of Section 2.3.6.

Needless to say, such a conception of the universe dates back to the very birth of cosmology, although it was abandoned due to various reasons, including the lack of a consistent classical theory of antimatter, inconsistencies for negative energies, and other problems.

The above conception of the universe was then replaced with the “big bang” conjecture implying a huge discontinuity at creation, in which a possible antimatter component in the universe is essentially left untreated.

All the above problems are resolved by the isodual theory of antimatter, and quantitative astrophysical studies on antimatter galaxies and quasars can now be initiated at the purely classical level.

Moreover, the prediction that the *isodual light* emitted by antimatter experiences a repulsion in the gravitational field of matter [38], permits the initiation of actual measurements on the novel *antimatter astrophysics.*

Noticeably, there already exist reports that certain astrophysical events can only be explained via the repulsion experiences by light emitted by certain galaxies or quasars, although such reports could not be subjected to due scientific process since the mere existence of such a repulsion would invalidate Einstein’s gravitation, as studied in Section 1.4.

Even though the assumption of an equal distribution of matter and antimatter in the universe dates back to the discovery of antimatter itself in the early 1930s, the Iso-Self-Dual Cosmology is structurally new because it is the first cosmology in scientific records based on a *symmetry,* let alone an *isoselfdual symmetry,* that of Dirac’s equation subjected to isotopies, Eqs. (14.1.43), i.e.,

$$\hat{S}_{Tot} = (\hat{\mathcal{P}}(3.1) \hat{\times} \hat{G}) \times (\hat{\mathcal{P}}(3.1)^d \hat{\times}^d \hat{G}^d) =$$
$$= [\hat{SL}(2,\hat{C})\hat{\times}\hat{T}(3.1)\hat{\times}\hat{\mathcal{I}}(1)] \times [\hat{SL}^d(2,\hat{C}^d)\hat{\times}^d\hat{T}^d(3.1)\hat{\times}^d\hat{\mathcal{I}}^d(1)]. \tag{14.2.1}$$

In fact, virtually all pre-existing cosmologies are based on Einstein’s gravitation, thus eliminating a universal symmetry *ab initio.*

Other novelties of the Iso-Self-Dual Cosmology are given by the implications, that are impossible without the isotopies and isodualities, such as:

1) The direct interpretation of the expansion of the universe, as well as the increase of the expansion itself, since antigravity is permitted by the isodualities but not in general by other theories;

2) The prediction that the universe has absolutely null total characteristics, that is, an absolutely null total time, null total mass, null total energy, null total entropy, etc., as inherent in all isoselfdual states[8];

3) The creation of the universe without any discontinuity at all, but via the joint creation of equal amounts of matter and antimatter, since all total characteristics of the universe would remain the same before and after creation.

We also mention that the isoselfdual cosmology was proposed by Santilli [44] to initiate mathematical and theoretical studies on the creation of the universe,

[8]We are here referring to intrinsic characteristic of isoselfdual states, and not to the same characteristics when inspected from a matter or an antimatter observer that would be evidently impossible for the universe.

studies that are evidently prohibited by theories with huge discontinuities at creation.

After all, we should not forget that the Bible states the creation first of light and then of the universe, while it is now known that photons can create a pair of a particle and its antiparticle.

Also, there is a mounting evidence that space (the *aether* or the *universal substratum*) is composed of a superposition of positive and negative energies, thus having all pre-requisites needed for the creation of matter and antimatter galaxies.

As one can see, a very simple property of the new number theory, the invariance under isoduality as it is the case for the imaginary unit (Section 2.1.1),

$$i \equiv i^d = -i^\dagger = -\bar{i}, \tag{14.2.2}$$

acquires a fundamental physical character for a deeper understanding of Dirac's gamma matrices (Chapter 2),

$$\gamma_\mu \equiv \gamma_\mu^d = -\gamma_\mu^\dagger, \tag{14.2.3}$$

and then another fundamental character for the entire universe.

To understand the power of isodualities despite their simplicity, one should meditate a moment on the fact that the assumed main characteristics of the universe as having an equal amount of matter and antimatter, can be reduced to a primitive abstract axiom as simple as that of the new invariance (14.2.2).

Needless to say, the condition of exactly equal amounts of matter and antimatter in the universe is a *limit case,* since in reality there may exist deviations, with consequential *breaking of the isoselfdual symmetry* (14.2.1). This aspect cannot be meaningfully discussed at this time due to the abyssal lack of knowledge we now have on the antimatter component in our universe.

It should be finally indicated that, in view of the topological features assumed for the basic isounit

$$\hat{I} = \hat{I}^\dagger > 0, \tag{14.2.4}$$

the Iso-Self-Dual Cosmology outlined above can only represent a closed and reversible universe, thus requiring suitable broadening for more realistic theories.

Recall that, from its Greek meaning, "cosmology" denotes the entire universe. Consequently, no theory formulated until now, including the Iso-Self-Dual Theory, can be called, strictly speaking, a "cosmology" since the universe is far from being entirely composed of closed and reversible constituents.

To begin, there is first the need to represent irreversibility, since the behavior in time of all stars, galaxies and quasars in the universe is indeed irreversible.

This first need can be fulfilled with the Iso-Self-Dual Cosmology realized via isounits that are positive-definite, but explicitly time dependent,

$$\hat{I}(t,\dots) = \hat{I}^{\dagger}(t,\dots) \neq \hat{I}(-t,\dots), \tag{14.2.5}$$

which feature assures irreversibility, although the universe remains closed due to the conservation of the total energy of matter and that of antimatter.

The latter model has evident limitations, e.g., in view of the possible continuous creation of matter and antimatter advocated by various researchers as an alternative to the "big bang".

The latter condition, when joint with the necessary representation of irreversibility, requires the broader *Geno-Self-Dual Cosmology*, namely, a cosmology based on the Lie-admissible lifting of symmetry (14.2.1), via the further generalization of generalized units (14.3.4) and (14.2.5) into four genounits, one per each of the four possible directions of time

$$\hat{I}^{>}, \quad -\hat{I}^{>}, \quad \left(\hat{I}^{>}\right)^{d} = -{}^{<}\hat{I}, \quad -\left(\hat{I}^{>}\right)^{d} ={}^{<}\hat{I}, \tag{14.2.6}$$

whose explicit construction is left to the interested reader for brevity (see Chapter 5).

Nevertheless, the latter genotopic lifting itself cannot be considered, strictly speaking, a "cosmology" because a basic component of the universe is life, for which genotopic theories are insufficient, as indicated in Section 3.7, due to their single-valuedness.

The latter need inevitably requires the formulation of cosmologies via the most general possible methods studied in this monograph, the multivalued hyperstructure of Chapter 5, resulting in the *Hyper-Self-Dual Cosmology,* namely, a cosmology based on the hyperlifting of symmetry (14.2.1) characterized by the ordered multivalued hyperunits

$$\hat{I}^{>} = \{\hat{I}_1^{>}, \hat{I}_2^{>}, \hat{I}_3^{>}, \dots\}, \quad -\hat{I}^{>} = \{-\hat{I}_1^{>}, -\hat{I}_2^{>}, -\hat{I}_3^{>}, \dots\}, \tag{14.2.7a}$$

$$(\hat{I}^{>})^{d} = \{-{}^{<}\hat{I}_1, -{}^{<}\hat{I}_2, -{}^{<}\hat{I}_3, \dots\}, \quad -(\hat{I}^{>})^{d} = \{{}^{<}\hat{I}_1, {}^{<}\hat{I}_2, {}^{<}\hat{I}_3, \dots\}. \tag{14.2.7b}$$

However, at this point we should remember the limitations of our mind and admit that the foundations of the Hyper-Self-Dual Cosmology, such as the multi-valued hypertime encompassing all four directions of time, is simply beyond our human comprehension.

After all, we have to admit that a final scientific understanding of life will likely require thousands of years of studies.

14.3 CONCLUDING REMARKS

The analysis conducted in this monograph establishes that the isodual theory of antimatter does indeed resolve the scientific imbalance of the 20-th century caused by the treatment of matter at all levels of study, and the treatment of antimatter at the sole level of second quantization.

In fact, the isodual theory of antimatter achieves an absolute democracy of treatment of both matter and antimatter at all levels, from Newton to second quantization.

In particular, the analysis presented in this monograph establishes that the isodual theory of antimatter is verified by all known experimental data on antimatter, since the isodual theory trivially represents all available classical experimental data (Section 2.2.3), while resulting in being equivalent to charge conjugation at the operator level (Section 2.3.7), as a result of which the entire currently available experimental knowledge on antiparticles is verified by the isodual theory.

Despite its simplicity, the isodual theory of antimatter has deep implications for all quantitative sciences, including classical mechanics, particle physics, superconductivity, chemistry, biology, astrophysics and cosmology.

The most salient consequence of the isodual theory is the prediction of antigravity experienced by *elementary* antiparticles in the field of matter and vice-versa.

This prediction is a direct consequence of the very existence of a consistent classical formulation of antimatter, the electromagnetic origin of the gravitational mass with consequential phenomenological equivalence of electromagnetism and gravitation for both attraction and repulsion, the forgotten Freud identity of the Riemannian geometry, and other aspects.

In reality, the prediction of antigravity for truly elementary antiparticles in the field of matter is rooted in so many diversified aspects that the possible experimental disproof of antigravity would likely require the reconstruction of theoretical physics from its foundations.

To minimize controversies, it should be stressed that the prediction of antigravity has been solely and specifically presented for *elementary* antiparticles, that is, for the *positron,* with the careful exclusion for first tests of any unstable or composite particles whose constituents are not seriously established as being all antiparticles.

As an illustration, we have discouraged the use in possible experiments on the gravity of the positronium as claim for final knowledge on the gravity of antimatter, because the positronium is predicted by the isodual theory to be attracted in both fields of matter and antimatter. Similarly we have discouraged the use of leptons because they may eventually result to be composite of particles and antiparticles.

Finally, we have strongly discouraged to assume experimental data on the gravity of antiprotons as final knowledge on the gravity of antiparticles, because antiprotons are today fabricated in high energy laboratories from matter components and are believed to be bound states of quarks for which no gravity at all can be consistently defined [38].

It then follows that, while all experimental data are indeed useful and should be supported, including experimental data on the gravity of antiprotons, their use for general claims on the gravity of antimatter could be deceptive.

Moreover, none of the numerous arguments against antigravity could even be properly formulated for the isodual theory, let alone have any value. As a result, the prediction of antigravity for elementary antiparticles in the field of matter is fundamentally unchallenged at this writing on theoretical grounds.

A test of the gravity of positrons in horizontal flight in a vacuum tube, that is resolutory via gravitational deflections visible to the naked eye, has been proposed by Santilli [45] and proved by the experimentalist Mills [46] to be feasible with current technology and be indeed resolutory (Section 4.2).

A comparative study of other tests has revealed that they are too delicate and require too sensitive measurements to be as resolutory as proposal [45] with current technologies.

It is hoped that the experimental community finally comes to its senses, and conducts fundamental test [45,46], rather than continuing to conduct tests of transparently less relevance at bigger public costs, because in the absence of a final experimental resolution of the problem of antigravity, the entire theoretical physics remains essentially in a state of suspended animation.

In turn, the possible experimental verification of antigravity (as above identified) would have implications so advanced as to be at the edge of our imagination.

One of these implications has been presented in Section 13.3 with the Causal Time Machine, the novel, non-Newtonian *isolocomotion* (propulsion to unlimited speeds without any action and reaction as requested by all currently available propulsions), and other far reaching possibilities.

The experimental resolution of the existence of antigravity for *truly elementary* antiparticles is also crucial to fulfil the original scope for which the isodual theory was built, namely, to conduct quantitative studies as to whether far-away galaxies and quasars are made up of matter or antimatter.

This main scope has been achieved via the *isodual photon,* namely, the discovery that, according to the isodual theory, photons emitted by antimatter appear to have a number of physical differences with the photons emitted by matter. In particular, *the simplest possible isodual electromagnetic waves have negative energy, thus experiencing antigravity in the field of matter.*

The above prediction requires the experimental resolution as to whether light emitted by antimatter is attracted or repelled by the gravitational field of matter.

Needless to say, the current availability at CERN of the antihydrogen atom is an ideal source for such a study, with the understanding that gravitational deflections of light at short distances (as attainable in a laboratory on Earth) are extremely small, thus implying extremely sensitive measurements.

More promising is the re-inspection of available astrophysical data privately suggested to the author because said data could already include evidence of light from far-away galaxies and quasars that is repelled by astrophysical objects closer to us.

Such a repulsion could not be publicly disclosed at this time because of known opposition by organized academic interests on Einsteinian doctrines since, as well known, Einstein's gravitation prohibits the existence of antigravity (Section 4.1).

It is hoped that such organized academic interests come to their senses too, if nothing else, to avoid an easily predictable serious condemnation by posterity, in view of the well known catastrophic inconsistencies of Einstein gravitation outlined in Section 1.4.

After all, we should not forget that antiparticles were first experimentally detected in cosmic rays, thus confirming their possible origin from supernova explosions of stars made up of antimatter.

Also, there are reports of huge explosions in Earth's atmosphere before the advent of atomic bombs without any crater on the ground, such as the 1908 Tunguska explosion in Siberia, which explosions can be best interpreted as antimatter asteroids from far away antimatter galaxies or quasars penetrating in our atmosphere.

Therefore, it should not be surprising if light experiencing gravitational repulsion from matter is discovered first in astrophysics.

Additional tests on the possible gravitational repulsion of light emitted by antimatter can be done via the direct measurement of the deflection of light from far away galaxies and quasars when passing near one of the planets of our Solar system.

Under the assumption of using light originating from far away galaxies and quasars (to render plausible their possible antimatter nature), and for the use of a sufficient number of galaxies and quasars (to have a sufficient probability that at least one of them is made up of antimatter), these astrophysical measurements are potentially historical, and will signal the birth of the new science proposed in this monograph under the name of *antimatter astrophysics.*

The reader should be aware that, while the prediction of antigravity for *truly elementary* antiparticles is an absolute necessity for the validity of the isodual theory, the gravitational behavior of light emitted by antimatter is not that simple.

Recall from Section 13.2 that the prediction of antigravity for light emitted by antimatter is based on the negative value of its energy for the selected solution of the electromagnetic wave.

However, the photons is invariant under charge conjugation and travel at the maximal causal speed in vacuum, c. Therefore, the photon could well result to be a superposition of positive and negative energies, perhaps as a condition to travel at the speed c, in which case the photon would be an isoselfdual state, thus experiencing attraction in both fields of matter and antimatter.

As a consequence, the possible disproof of antigravity for light emitted by antimatter stars in the field of matter *would not* invalidate the isodual theory of antimatter, but merely tell us that our conception of light remains excessively simplistic to this day, since it could well be in reality a composite state of photons and their isoduals.

The issue is further complicated by the fact indicated during the analysis of this monograph that *antigravity is predicted between masses with opposite time evolutions,* as it is the case for a positron in the field of Earth. However, the photon travels at the speed of light at which speed time has no meaningful evolution.

As a result, it is not entirely clear to this author whether the sole value of negative energy for the isodual light is sufficient for the existence of a gravitational repulsion, and the issue is suggested for study by interested colleagues.

To express a personal view, it would be distressing if light solely experience gravitational attraction irrespective of whether in the field of matter or antimatter and whether originating from matter or antimatter, because this would imply the impossibility for experimental studies as to whether far-away galaxies and quasars are made up of matter or antimatter, since all other aspects, including thermodynamics, are not detectable at large distances, thus implying the perennial inability for mankind to reach any in depth knowledge of the universe.

The author does not believe so. Advances in human knowledge have no limit, and often go beyond the most vivid imagination, as established by scientific realities that resulted in being beyond the science fiction of preceding generations.

In closing, the author hopes that the studies presented in this monograph have stimulated young minds of any age and confirmed that science will never admit final theories. No matter how precious, beloved and valid a given theory may appear to be at a given time, its surpassing with broader theories more adequate for new scientific knowledge is only a matter of time.

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