Toward a Traversible Wormhole

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Abstract. In this talk I discuss pertinence of the wormholes to the problem of circumventing the light speed barrier and present a specific class of wormholes. The wormholes of this class are static and have arbitrarily wide throats, which makes them traversable. The matter necessary for these spacetimes to be solutions of the Einstein equations is shown to consist of two components, one of which satisfies the Weak energy condition and the other is produced by vacuum fluctuations of neutrino, electromagnetic (in dimensional regularization), and/or massless scalar (conformally coupled) fields.

WORMHOLES AND THEIR RELATION TO HYPER-FAST TRAVEL

Wormholes are geometrical structures connecting two more or less flat regions of a spacetime. This of course is not a rigorous definition, but, strange though it may seem, there is no commonly accepted rigorous definition of the wormhole yet. Normally, however, by a wormhole a spacetime is understood resembling that obtained by the following manipulation:

1) Two open balls are removed each from a piece of approximately flat 3-space (the vicinities of thus obtained holes we shall call *mouths* of the wormhole);
2) The boundaries (2-spheres) of the holes are glued together, and the junction is smoothed. In the process of smoothing a kind of tube arises interpolating the spheres. We shall call this tube the *tunnel* and its narrowest part the *throat*.

The resulting object (its two-dimensional version to be precise) is depicted in Fig.1. If in the course of evolution the spacetime surrounding such an object remains approximately flat (which may not be the case, since flatness of each 3-dimensional section does not guarantee that the 4-dimensional space formed by them is also flat) we shall call the object a wormhole.

Wormholes arise in a natural way in general relativity. Even one of the oldest and best-studied solutions of the Einstein equations — the Schwarzschild spacetime — contains a wormhole, which was found at least 80 years ago (Flamm, 1916). This wormhole (also known as the Einstein-Rosen bridge) connects two asymptotically flat regions ('two universes'), but being non-static is useless in getting from one of them to the other (see below).

Depending on whether the tunnel connects distant regions of 'the same' universe or these regions are otherwise not connected the wormholes fall into two categories (Visser, 1995). To which category a wormhole belongs depends on how the vicinities of the mouths are extended to the full spacetime:

It may happen that the mouths cannot be connected by any curve except those going through the tunnel (as it takes place in the Einstein-Rosen bridge). Such wormholes are called *inter-universe*. A simplest static spherically symmetric inter-universe wormhole can be described (Morris, 1988) by a manifold $R^2 \times S^2$ endowed with the metric

$$ds^2 = -e^{2\Phi} dt^2 + dr^2/(1-b/r) + r^2(\mathrm{d}\theta^2 + \sin^2\theta\, d\phi^2), \tag{1}$$
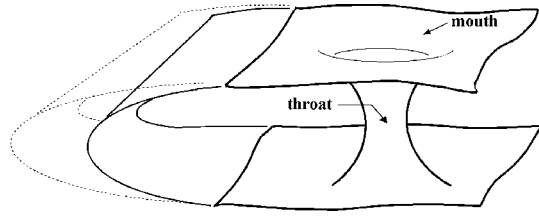
FIGURE 1. The sketch of a wormhole with the mouths in motion. One dimension (corresponding to the coordinate ϑ) is omitted. The ways in which the upper and the lower parts are glued at $t = 0$ and at $t = 1$ are depicted by thin solid lines and by dashed lines respectively. Though the geometry of the wormhole does not change, the distance (as measured in the outer, flat space) between mouths increases with time.

where $r \in (-\infty, \infty)$ (note this possibility of negative r, it is the characteristic feature of the wormholes), $\Phi(r) \to 0$ and $b(r)/r \to 0$, when $r \to \pm\infty$.

Alternatively as shown in Fig.1 it may happen that there are curves from one mouth to another lying outside the wormhole. Such a wormhole connects distant parts of a 'single' universe and is called *intra-universe*. Though intra-universe wormholes are in a sense more interesting most papers deal with inter-universe ones, since they are simpler. It does not matter much, however. The distant regions of the 'universes' are taken to be approximately flat. And it is usually implied that given an inter-universe wormhole we can as well build an intra-universe one by simply gluing these distant regions in an appropriate way.

It is stable intra-universe wormholes that are often used for interstellar travel in science fiction (even though they are sometimes called 'black holes' there). Science fiction (especially Sagan's novel *Contact*) has apparently acted back on science and in 1988 Morris and Thorne pioneered investigations (Morris, 1988) of what they called *traversible wormholes* — wormholes that can be (at least in principle) traversed by a human being. It is essential in what follows that to be traversible a wormhole should satisfy at least the following conditions:

(C1). It should be sufficiently long-lived. For example the Einstein-Rosen bridge connects two asymptotically flat regions (and so it is a wormhole), but it is not traversible — the throat collapses so fast that nothing (at least nothing moving with $v \leq c$) can pass through it.

(C2). It should be *macroscopic*. Wormholes are often discussed [see (Hochberg, 1997), for example] with the radius of the throat of order of the Plank length. Such a wormhole might be observable (in particular, owing to its gravitational field), but it is not obvious (and it is a long way from being obvious, since the analysis would inevitably involve quantum gravity) that any signal at all can be transmitted through its tunnel. Anyway such a wormhole is impassable for a spaceship.

Should a traversible wormhole be found it could be utilized in interstellar travel in the most obvious way. Suppose a traveler (say, Ellie from the above-mentioned novel) wants to fly from the Earth to Vega. One could think that the trip (there and back) will take at least 52 years for her even if she moves at a nearly light speed. But if there is a wormhole connecting the vicinities of the Earth and Vega she can take a short-cut by flying through it and thus make the round trip to Vega in (almost) no time.

Note, however, that such a use of a wormhole would have had nothing to do with circumventing the light barrier. Indeed, suppose that Ellie's start to Vega is appointed on a moment $t = 0$. Our concern is with the time interval Δt_E in which she will return to the Earth. Suppose that we know (from astronomical observations, theoretical calculations, etc.) that if in $t = 0$ she (instead of flying herself) just emit a photon from the Earth, this photon after reaching Vega (and, say, reflecting from it) will return back at best in a time interval Δt_P. If we find a wormhole from the Earth to Vega, it would only mean that Δt_P actually is small, or in other words that Vega is actually far closer to the Earth than we think now. But what can be done if Δt_P *is* large (one would hardly expect that traversible wormholes can be found for *any* star we would like to fly to)? That is where the need in hyper-fast transport comes from. In other words, the problem of circumventing the light barrier (in connection with interstellar travel) lies in the question: how to reach a remote (i. e. with the large Δt_P) star and to return back sooner than a photon would have made it (i. e. in $\Delta t_E < \Delta t_P$)? It makes sense to call a spaceship faster-than-light (or *hyper-fast*) if it solves this prolem.

A possible way of creating hyperfast transport lies also in the use of traversible wormholes (Krasnikov, 1998). Suppose that a traveler finds (or builds) a traversible wormhole with both mouths located near the Earth and suppose that she can move the mouths (see Fig.1) at will without serious damage to the geometry of the tunnel (which we take to be negligibly short). Then she can fly to Vega taking one of the mouths with her. Moving (almost) at the speed of light she will reach Vega (almost) instantaneously by her clocks. In doing so she *rests* with respect to the Earth insofar as the distance is measured through the wormhole. Therefore her clocks remain synchronous with those on the Earth as far as this fact is checked by experiments confined to the wormhole. So, if she return through the wormhole she will arrive back to the Earth almost immediately after she will have left it (with $\Delta t_E << \Delta t_P$).

Remark 1. The above arguments are very close to those showing that a wormhole can be transformed into a time machine (Morris, 1988), which is quite natural since the described procedure is in fact the first stage of such transformation. For, suppose that we move the mouth back to the Earth reducing thus the distance between the mouths (in the ambient space) by 26 light years. Accordingly Δt_E would lessen by ≈ 26 yr and (being initially very small) would turn *negative*. The wormhole thus would enable a traveller to return before he have started. Fortunately, $\Delta t_E \approx 0$ would fit us and we need not consider the complications (possible quantum instability, paradoxes, etc.) connected with the emergence of thus appearing time machine.

Remark 2. Actually two *different* worlds were involved in our consideration. The geometry of the world where only a photon was emitted differs from that of the world where the wormhole mouth was moved. A photon emitted in $t = 0$ in the latter case would return in some $\Delta t_{P'} < \Delta t_E$. Thus what makes the wormhole-based transport hyper-fast is changing (in the causal way) the geometry of the world so that to make $\Delta t_{P'} < \Delta t_E << \Delta t_P$.

Thus we have seen that a traversible wormhole can possibly be used as a means of 'superluminal' communication. True, a number of serious problems must be solved before. First of all, where to get a wormhole? At the moment no good recipe is known how to make a *new* wormhole. So it is worthwhile to look for 'relic' wormholes born simultaneously with the Universe. Note that though we are not used to wormholes and we do not meet them in our everyday life this does not mean by itself that they are an exotic rarity in nature (and much less that they do not exist at all). At present there are no observational limits on their abundance [see (Anchordoqui, 1999) though] and so it well may be that there are 10 (or, say, 10^6) times as many wormholes as stars. However, so far we have not observed any. So, this issue remains open and all we can do for the present is to find out whether or not wormholes are allowed by known physics.

CAN TRAVERSIBLE WORMHOLES EXIST?

Evolution of the spacetime geometry (and in particular evolution of a wormhole) in general relativity is determined via the Einstein equations by properties of the matter filling the spacetime. This circumstance may turn out to be fatal for wormholes if the requirements imposed on the matter by conditions (C1,C2) are unrealistic or conflicting. That the problem is grave became clear from the very beginning: it was shown (Morris, 1988), see also (Friedman, 1993), that under very general assumptions the matter filling a wormhole must violate the Weak Energy Condition

(WEC) The WEC is the requirement that the energy density of the matter be positive in any reference system. When the stress-energy tensor T_{ik} is diagonal the WEC may be written as

WEC: $\qquad\qquad\qquad\qquad T_{00} \geq 0, \quad T_{00} + T_{ii} \geq 0, \qquad i = 1, 2, 3$ $\qquad\qquad\qquad$ (2)

Classical matter always satisfies the WEC (hence the name '*exotic*' for matter violating it). So, a wormhole can be traversible only if it is stabilized by some quantum effects.

Candidate effects are known, indeed [quantum effects can violate any local energy condition (Epstein, 1965)]. Moreover, owing to the non-trivial topology a wormhole is just a place where one would expect WEC violations due to fluctuations of quantum fields (Khatsymovsky, 1997a). So, the idea appeared (Sushkov, 1992) to seek a wormhole with such a geometry that the stress-energy tensor produced by vacuum polarization is exactly the one necessary for maintaining the wormhole. An example of such a wormhole (it is a Morris-Thorne spacetime filled with the scalar non-minimally coupled field) was offered in (Hochberg, 1997). Unfortunately, the diameter of the wormhole's throat was found to be of the Plank scale, that is the wormhole is non-traversible.

The situation considered in (Hochberg, 1997) is of course very special (a specific type of wormholes, a specific field, etc.). However arguments were cited [based on the analysis of another energetic condition, the so called ANEC (Averaged Null Energy Condition)] suggesting that the same is true in the general case as well (Flanagan, 1996, see also the literature cited there). So an impression has been formed that conditions (C1) and (C2) are incompatible, and TWs are thus impossible.

YES, IT SEEMS THEY CAN

The question we are interested in is whether such macroscopic wormholes exist that they can be maintained by the exotic matter produced by the quantum effects. To put it more mathematically let us first separate out the contribution T^Q_{ik} of the 'zero-point energy' to the total stress-energy tensor T_{ik}:

$$T_{ik} = T^Q_{ik} + T^C_{ik}. \qquad\qquad\qquad (3)$$

In semiclassical gravity it is deemed that for a field in a quantum state $|\Psi\rangle$ (in particular, $|\Psi\rangle$ may be a vacuum state) $T^Q_{ik} = \langle\Psi| \, \boldsymbol{T}_{ik} \, |\Psi\rangle$, where $\boldsymbol{T}_{ik}$ is the corresponding operator, and there are recipes for finding T^Q_{ik} for given field, metric, and quantum state [see, for example, (Birrel, 1982)]. So, in formula (3) T^Q_{ik} and T_{ik} are determined by the geometry of the wormhole and the question can be reformulated as follows: do such macroscopic wormholes exist that the term T^C_{ik} describes usual non-exotic matter, or in other words that T^C_{ik} satisfies the Weak Energy Condition, which now can be written as

$$G_{00} - 8\pi T^Q_{00} \geq 0, \quad (G_{00} + G_{ii}) - 8\pi(T^Q_{00} + T^Q_{ii}) \geq 0, \qquad i = 1, 2, 3. \qquad (4)$$

(we used the formulas (2,3) here)?

One of the main problem in the search for the answer is that the relevant mathematics is complicated and unwieldy. A possible way to obviate this impediment is to calculate T^Q_{ik} numerically (Hochberg, 1997; Taylor 1997) using some approximation. However, the correctness of this approximation is in doubt (Khatsymovsky, 1997b), so we shall not follow this path. Instead we shall study a wormhole with such a metric that relevant expressions take the form simple enough to allow the analytical treatment.

The Morris-Thorne wormhole is not the unique static spherically symmetric wormhole (contrary to what can often be met in the literature). Consider a spacetime $R^2 \times S^2$ with the metric:

$$ds^2 = \Omega^2(\xi) \, [-d\tau^2 + d\xi^2 + K^2(\xi)(d\theta^2 + \sin^2\theta \, d\phi^2)], \qquad (5)$$

where Ω and K are smooth positive even functions, $K = K_0 \cos\xi/L$ at $\xi \in (-L, L)$, $K_0 \equiv K(0)$ and K is constant at large ξ. The spacetime is obviously spherically symmetric and static. To see that it has to do with wormholes consider the case

$$\Omega \sim \Omega_0 \exp\{B\xi\}, \qquad \text{at large } \xi. \tag{6}$$

The coordinate transformation

$$r \equiv B^{-1}\Omega_0 \exp\{B\xi\}, \qquad t \equiv Br\tau, \tag{7}$$

then brings the metric (5) in the region $t < r$ into the form:

$$ds^2 = -dt^2\, 2\, t/r\mathrm{dt}\, \mathrm{dr} + [1-(t/r)^2\,]dr^2 + (BK_0\,\mathrm{r})^2\,(\mathrm{d}\theta^2 + \sin^2\theta\, d\phi^2). \tag{8}$$

It is obvious from (7) that as r grows the metric (5,8) becomes increasingly flat (the gravitational forces corresponding to it fall as $1/r$) in a layer $|t| < T$ (T is an arbitrary constant). This layer forms a neighborhood of the surface $\tau \equiv t \equiv 0$. But the spacetime is static (the metric does not depend on τ). So, the same is true for a vicinity of *any* surface $\tau \equiv const$. The spacetime can be foliated into such surfaces. So this property (increasing flatness) holds in the whole spacetime, which means that it is a wormhole, indeed. Its length (the distance between mouths as measured through the tunnel) is of order of $\Omega_0 L$ and the radius of its throat $R_0 = \min(\Omega K)$.

The advantage of the metric (5) is that for the electro-magnetic, neutrino, and massless conformally coupled scalar fields $T^Q{}_{ik}$ can be readily found (Page, 1982) in terms of Ω, K and their derivatives [actually the expression contains also one unknown term (the value of $T^Q{}_{ik}$ for $\Omega = 1$), but the more detailed analysis shows that for sufficiently large Ω this term can be neglected]. So, by using this expression, calculating the Einstein tensor G_{ik} for the metric (5) and substituting the results into the system (4) we can recast it [the relevant calculations are too laborious to be cited here (the use of the software package *GRtensorII* can lighten the work significantly though)] into the form:
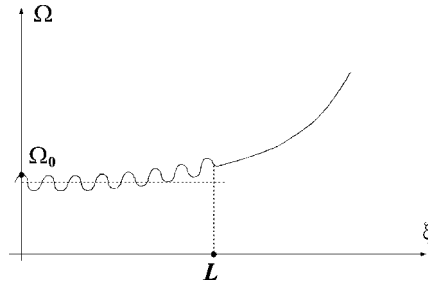

FIGURE 2. A conformal factor Ω satisfying requirements (i) – (iii).

$$E_i \geq 0 \qquad i = 0, 1, 2, 3, \tag{9}$$

where E_i are some (quite complex, *e. g.* E_0 contains 40 terms; fortunately they are not all equally important) expressions containing Ω, K, and their derivatives and depending on what field we consider.

Thus if we restrict ourselves to wormholes (5), then to answer the question formulated above all we need is to find out whether such Ω exist that it
i)   has appropriate asymptotic behavior [see (6)],
ii) satisfies (9) for some field,
iii) delivers sufficiently large R_0.
It turns out (Krasnikov, 1999) that for all three fields listed above and for arbitrarily large R_0 such Ω do exist (an example is sketched in Fig.2) and so the answer is positive.

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