Warp Drive Today: theory and limitations

E. F. Halerewicz, Jr.
email: hal_warp@hotmail.com

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Abstract

A discussion is given on the theory of the 'warp drive,' which is explored from its FTL origins to its 'energy problems.'

1 The Warp Drive

"*Why are we now traveling into space? Why indeed do we trouble to look past the next mountain? Our prime obligation to ourselves is to make the unknown known. We are on a journey to keep an appointment with whatever we are .*"

–Gene Roddenberry (1921–1991)

In science-fiction exceeding the speed of light is fairly easy, all that is needed is a propulsion device capable of exceeding the speed of light. From Einstein's theory of Special Relativity (SR), we are lead to believe nothing can go faster than the speed of light, thus Faster Than Light (FTL) travel is simply not possible. However, this is a miss reading of Lorentz-Fitzgerled

length contraction and time dilation, which by definition only applies to masses by

$$E = m + \frac{m}{2}q^2 + \frac{3}{8}\dots. \tag{1}$$

Thus the light barrier need only apply for bodies with a corresponding 'mass-energy' content, therefore the spatial coordinates have no such limit. From SR many physicist seem to wrongly assume the speed for gravitational radiation is the speed of light c. After all if the Sun were to mysteriously disappear we wouldn't know for another eight minutes, otherwise causality is violated, and time paradoxes pop up. However, this logic only applies to a strict four-dimensional spacetime, this line of logic no longer hold in the dimensions are greater, e.g. *superstring theory*. Furthermore, the infamous EPR-Bell thought experiment would seem to imply that there is some strict restrictions on this requirement, but the purpose of this paper is not to dwell into quantum mechanics. In short it is possible within General Relativity (GR), to exceed the speed of light through a special case coordinate transformation. The so called light barrier is only a weak definition and in principle can be broken, such a special case was given by Miguel Alcubierre in 1994 [1]. This form of propulsion was envisioned first within the science-fiction genera, which we know is the *Warp Drive* of *Star Trek*.

2 Alcubierre's clever idea

In the last section, we can see why many in scientific community would shrug at apparent FTL travel at a first glance. But as we have seen earlier it is the intuition of the first postulate of SR which brings this reasoning on which is of course a weak definition of equation (1), which doesn't apply for GR. In order to see this one must curve the flat Minkowski space to the following order $ds^2 = -dct^2 + dx^2 + dy^2 + dz^2$. Let us assume an arbitrary function of x, i.e. $f(x)$ so that we have $ds^2 = -dt + (f(x))^2 + dy^2 + dz^2$ in Minkowski spacetime. In the case of Alcubierre's Warp Drive we have a specific metric signature (or curvature) which is time dependent of form:

$$ds^2 = -dt^2 + (dx - v_s f(r_s)dt)^2 + dy^2 + dz^2 \tag{2}$$

where

$$v_s(t) = \frac{dx_s(t)}{dt} \tag{3}$$

which is simply the velocity of the system given through calculus as a function of time. The invariant spatial radius r_s is defined as

$$r_s(t) = [(x - x_s(t))^2 + y^2 + z^2]^{1/2}. \tag{4}$$

This spacetime model is often referred to a 'top hat' model, by means of a high energy density σ (which is the energy that must be applied to bend space (this also forms the "wall" of the warp bubble)), which is seen through

$$f(r_s) = \frac{\tanh(\sigma(r_s + R)) - \tanh(\sigma(r_s - R))}{2\tanh(\sigma R)} \tag{5}$$

This equation then allows for a "warp bubble" to develop in which a *star ship* may ride. These generalities can be described by the trigonometric expression

$$\theta = v_s \frac{x_s}{r_s} \frac{df}{dr_s}, \tag{6}$$

which also gives the spacetime its characteristic 'negative energy'(it is possible that a new top-hat could reduce the energy requirements all together, but this is not the topic of this paper). if graphed in the standard x-y-z plane the spaital coordinates yield:

$$x \quad and \qquad \rho = (y^2 + z^2)^{1/2}. \tag{7}$$

In short, the warp drive works by **contracting the space in front of a ship, and expanding space at its rear**. If you compare the Alcubierre geometry with that of flat spacetime, you realize the beauty and simplicity of the theory, even Alcubierre noted this in his work.

2.1 traveling at warp speed

In science-fiction the effects of FTL are really quite simple, streak star light in front of camera. Well as you probably guessed this notion is entirely false, it does something much stranger. The Contracting of spacetime in front of the ship causes the wavelength of light to shorten, becoming blue shifted. So the faster you travel, the stars appear closer and closer to one another and seem to shift towards the color blue. However, looking backwards the stars seem to repel one another, and seem to be red shifted, much like the galaxies within our expanding Universe.

In theory, there exist another way in which warp drive might look, see section (4.1). Using a special kind of black hole, known as a wormhole a spacetime can be produced which can mimic the warp drive known as the *Krasnikov Tube*. In this spacetime model there is no special "warping" effect, space appears normal, however on a return trip this tube yields a short cut, making it appear as a special case of warp drive propulsion.

3 an attack on the warp drive

The problem behind warp theory resides with the violation of certain conservation laws within GR. Known specifically as the Weak Energy Condition (WEC), Strong Energy Condition (SEC), and Generic Energy Condition (GEC), since this paper does not put a strong emphasis on GR we will neglect defining these terms. The important fact to note (which Alcubierre was aware when he wrote his paper), is that the energy needed for the warp drive violates the WEC. In short, the violation of the WEC implies the existence of negative energies, which defies thermodynamics and the law of conservation of energy. Although negative energy is allowed through the loopholes of quantum mechanics it is forbidden with the classical conception of GR. In fact it requires an energy requirement which supersedes even the laws of quantum mechanics. In fact the energy is so large, that it drawfs all the energy in the observable universe. The minimum negative energy required for the warp drive was given by Ford and Pfenning [2], in terms of the radius of the "warp bubble":

$$E = -\frac{1}{12} v_s^2 \left(\frac{(R + \frac{\triangle}{2})^2}{\triangle} + \frac{\triangle}{12} \right) \tag{8}$$

For an area of one hundred meters this energy is $E \simeq -6.2 \times 10^{62} j$. From quantum mechanics the maximum amount of energy allowable for gravitation is given by the planck length $l_p = (G\hbar/c^3)^{1/2} \equiv 10^{34} j$, needless to say this amount of energy would seem impossible to produce. And this is why a vast majority of the scientific community proclaims that the warp drive is impossible to achieve. The reader should keep in mind that this is based of the most simplistic model for the warp drive, one which closely resembles the metric of flat spacetime, thus to kill the idea simply on the basis of this early metric appears premature.

3.1 the problem of control

We know that the idea of warp drive is possible, we know that GR does not rule out spacetime exceeding the "light barrier." But there is the little problem of maintaining and controlling the warp bubble. After one produces the so called "warp bubble," the contractory portion of the metric becomes casually disconnected with the wave function of the warp drive. This means that no matter how hard one tries to manipulated the bubble from within, all attempts will fail. It would only be possible to manipulate the bubble from outside, but how, well this question has no answer within the present research.

4 the second warp bubble

Needless to say, there have been numerous papers written on the 'impossibility' of warp drive, than in support of it. We will not speculate on the reason for this, but from the previous section it can be seen why many would be persuaded to follow this line of reasoning. However, there exist a way to lower the energy requirement for the Alcubierre metric for an area of 100 meters to $E_{-} = -1.4 \times 10^{30} j$. The difference in energy is a hundred thousand times a trillion times a trillion less than what is required by the original Alcubierre metric. The trick, is shrinking the warp bubble to microscopic proportion by a factor of $B^2(r_s)$. This is made possible by the work of Chris Van Den Broeck, through the metric [3]:

$$ds^2 = -dt^2 + B^2(r_s)[(dx - v_s f(r_s)dt)^2 + dy^2 + dz^2]. \tag{9}$$

For those of you familiar with GR the B term resembles the constant "C" used within the Schwartzschild metric (a spherical symmetric solution to the Einstein field equations). The trick of this spacetime metric is that it keeps the *surface area of the warp 'bubble' microscopic, while increasing its internal area by a factor of* $B^2(r_s)$.

4.1 FTL without warp drive

In the previous sections we have seen how the warp drive is possible, how it can be produced and its limitations. However, the generic term 'warp drive,' is not the only possible method for FTL travel. In Carl Sagan's

novel Contact, wormholes (a special case rotating massive black hole) were used for interstellar space travel. Wormholes are not forbidden within GR, however producing them seems impossible, you would likely have to find one to engage in interstellar travel. Kip Throne and others have shown that attempting to pass through a wormhole's "throat" would collapse it, unless of course one has a kind of negative energy field (this is where we see the connection to the warp drive). The next generation of warp drive is simply known as the **Krasnikov Tube**. This paper is even simpler in principle than the Alcubierre paper, because it deals with only one spatial dimension and one time dimension (which also makes it slightly less physical, in four spatial dimensions this space appears as a tube where the name is derived). Of course there is another little metric which goes along with this [4]:

$$ds^2 = -(dt - dx)(dt + k(t, x)dx) \tag{10}$$

where $k \equiv 1 - (2 - \delta)\theta_\epsilon(t - x)[\theta_\epsilon(x) - \theta_\epsilon(x + c - D)]$, θ_ϵ which represents a smooth function. This method of propulsion can be envisioned rather easily, by folding paper, there exist one Euclidean spacetime (original metric), and a second spacetime (i.e. the Kransnikov tube), and a pseudo spacetime corresponding to the smooth function, which is similar in appearance to the Alcubierre top-hat function $f(r_s)$.

5 quantum vacuum, and realization of negative energy

Although there are several problems facing the warp drive, the biggest challenge does not come from the concept itself. The biggest problem is the requirement of a tremendous amount of negative energy, this appears to be the Achilles heal of the theory. If one simply compares the consequences of negative energies whithin Einstein's formalism for gravitation we see that it is quite similar to the Cosmological Constant Λ[1]. Recently it has been proposed that the Cosmological Constant could be created by electromagnetic zero-point-field fluctuations, i.e. the quantum vacuum.

[1]Einstein in later years would call this pseudo prediction of his theory "the biggest blunder of my life." This is because the term would defy the attractive nature of gravity, by mysterious unknown force.

An idea pioneered by H. Puthoff is the idea that this "zero-point energy" could be tapped. This is not at all a new idea, however this runs along the idea of producing a fully functional perpetual motion machine (quite literally something from nothing). This idea alone itself may appear laughable, but that is before you consider the *Casimir Effect*. Which is a quantum force that can be felt by two parallel conducting metal plates. The net force acting on the plates is given through a quantum of energy of from $\frac{1}{2}h\omega$, or:

$$\frac{F(r)}{A} = \frac{\pi^2}{240}\frac{hc}{r^4}. \tag{11}$$

Where the strength of the force F is given by the distance r, over the area A of the applied force. Puthoff however, goes one step further going along with a theory that proposes gravitation is a consequence of this zero-point-energy [5]. This thus maybe seen as a way to get around the energy problem, however it would also appear that this would be no overnight solution.

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