

Spacetime Foam and Vacuum Energy

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A simple model of spacetime foam, made by spherically symmetric wormholes, with or without a cosmological term is proposed. The black hole area quantization and its consequences are examined in this context. We open the possibility of probing Lorentz symmetry in this picture.

1 Introduction

The term Spacetime Foam was introduced for the first time by J.A. Wheeler at the end of the fifties to indicate that quantum fluctuations come into play at the Planckian scale, changing topology and metric [1]. Apart the pioneering path integral approach to Spacetime Foam, considered by S.W. Hawking [2], only recently the subject has been widely reconsidered in the context of brane physics [3], spin foam models [4] and on a phenomenological ground [5]. We will briefly introduce a different approach based on a large N_w -wormhole semi-classical approximation. This model is motivated by the results on the computation of the Casimir energy computed on some wormhole backgrounds of spherical symmetry (Schwarzschild, Schwarzschild-de Sitter, Schwarzschild-Anti-de Sitter and Reissner-Nordström metrics) which have as a reference space the presumed ground state (Minkowski, de Sitter and Anti-de Sitter metrics) [6, 7, 8, 9]. The formulation of the Casimir effect in general is synthesized by the Eq.

$$E_{Casimir}[\partial\mathcal{M}] = E_0[\partial\mathcal{M}] - E_0[0], \quad (1)$$

where E_0 is the Zero Point Energy (Z.P.E.) and $\partial\mathcal{M}$ is a boundary. It is immediate to see that the Casimir energy involves a vacuum subtraction procedure and since this one is related to Z.P.E., we can extract information on the ground state. In concrete terms, we compute the following quantity

$$E(\text{wormhole}) = E(\text{no - wormhole}) + \Delta E_{\text{no-wormhole}|classical}^{\text{wormhole}} + \Delta E_{\text{no-wormhole}|1-loop}^{\text{wormhole}}, \quad (2)$$

representing the total energy computed to one-loop in a wormhole background. $E(\text{no - wormhole})$ is the reference space. $\Delta E_{\text{no-wormhole}|classical}^{\text{wormhole}}$ is the classical

energy difference between the wormhole and no-wormhole configuration stored in the boundaries and finally $\Delta E_{no-wormhole|1-loop}^{wormhole}$ is the quantum correction to the classical term. Usually, the last term in the examined metrics exhibits an unstable mode. In the Coleman language, we are perturbing the false vacuum [10]. Instead of rejecting such a configuration, we take into examination a large wormholes number.

2 Large N_w -wormhole approach to Spacetime foam

We consider N_w non-interacting wormholes in a semiclassical approximation and assume that there exists a covering of Σ such that $\Sigma = \cup_{i=1}^{N_w} \Sigma_i$, with $\Sigma_i \cap \Sigma_j = \emptyset$ when $i \neq j$. Each Σ_i has the topology $S^2 \times R^1$ with boundaries $\partial \Sigma_i^\pm$. Furthermore, we assume the existence of a bifurcation surface S_0 , which is the case for the wormhole background considered. On each surface Σ_i the energy stored in the boundaries is zero because we assume that on each copy of the single wormhole there is symmetry with respect to each bifurcation surface¹. In this context, the total energy contribution comes only by quantum fluctuations. Moreover, the instability appearing in the single wormhole case is eliminated in the multi-wormhole picture. As a consequence, the regularized total energy - Casimir energy - assumes the form, at its minimum,

$$\Delta E_{N_w}(\bar{x}) = -N_w \frac{V}{64\pi^2} \frac{\Lambda^4}{e} \quad (3)$$

valid even for the Schwarzschild-Anti-de Sitter case².

2.1 Some consequences of a foamy spacetime

Here we will consider some applications of the foam model. One consequence is a natural quantization process in terms of the wormholes constituents. In particular, the area of a black hole is examined in this context. The area is measured by the quantity

$$A(S) = \int_S d^2x \sqrt{\sigma}. \quad (4)$$

σ is the two-dimensional determinant coming from the induced metric σ_{ab} on the boundary S . The evaluation of the mean value of the area

$$A(S) = \frac{\langle \Psi_F | \hat{A} | \Psi_F \rangle}{\langle \Psi_F | \Psi_F \rangle} = \frac{\langle \Psi_F | \int_S \widehat{d^2x \sqrt{\sigma}} | \Psi_F \rangle}{\langle \Psi_F | \Psi_F \rangle}, \quad (5)$$

¹This can be proved in terms of quasilocal energy.

²The case of the Schwarzschild-de Sitter case gives for the Z.P.E. the following expression

$$\Delta E_{N_w}(\bar{x}) = -N_w \frac{V}{32\pi^2} \frac{\Lambda^4}{e}.$$

is computed on the following state

$$|\Psi_F\rangle = \Psi_1^\perp \otimes \Psi_2^\perp \otimes \dots \otimes \Psi_{N_w}^\perp. \quad (6)$$

Suppose to consider the mean value of the area A computed on a given *macroscopic* fixed radius R . On the basis of our foam model, we obtain $A = \bigcup_{i=1}^N A_i$, with $A_i \cap A_j = \emptyset$ when $i \neq j$. Thus $A = 4\pi l_p^2 N \alpha$. α represents how each single wormhole is distributed with respect to the black hole in terms of the area. When compared with the Bekenstein area spectrum proposal, α is fixed to $\ln 2/\pi$ and [11]

$$M = \frac{\sqrt{N}}{2l_p} \sqrt{\frac{\ln 2}{\pi}}, \quad (7)$$

namely the Schwarzschild black hole mass is *quantized* in terms of l_p . The level spacing of the transition frequencies is

$$\omega_0 = \Delta M = (8\pi M l_p^2)^{-1} \ln 2. \quad (8)$$

An interesting aspect comes when we consider Schwarzschild-Anti-de Sitter wormholes as foam constituents. Indeed, the level spacing of the transition frequencies is

$$\omega_0 = \Delta M_S^{AdS} = (8M_S l_p^2)^{-1} \frac{9 \ln 2}{16\pi} = \Delta M \frac{9}{16} \quad (9)$$

that it means that for a given Schwarzschild black hole of mass M , the S-AdS foam representation gives smaller frequencies. In this respect, we have found a possibility to understand which constituents form our foam model. The inclusion of a charge is straightforward and leads to

$$Q^2 = \sqrt{N_2} \left(\sqrt{N_1} - \sqrt{N_2} \right) \alpha l_p^2, \quad (10)$$

where N_2 is the wormholes number used for the covering of the RN black hole area. We immediately see that from the above equation we have $N_1 \geq N_2$, where the equality corresponds to the vanishing charge. Moreover we choose N_1 and N_2 in such a way that $Q^2 = \alpha l_p^2 q$, $q = 0, 1, 2, \dots$. This means that

$$\sqrt{N_2} \left(\sqrt{N_1} - \sqrt{N_2} \right) = q. \quad (11)$$

When $q = 0$, we recover the Schwarzschild case, namely $N_2 = N_1$. On the other hand, when we consider $N_1 = 4q$, we get $N_2 = q$ corresponding to the extreme case. This leads to the quantization of the Reissner-Nordström black hole mass

$$M = \frac{\sqrt{\alpha}}{2l_p} \sqrt{N_2} \left(1 + \frac{q}{N_2} \right). \quad (12)$$

When we consider a transition in mass with a fixed charge, on the level spacing we get

$$\omega_0 = \Delta M = \frac{\partial M}{\partial r_+} \frac{dr_+}{dN} \Delta N = \frac{\pi}{A} (r_+ - r_-) \alpha \quad (13)$$

with $\Delta N = 1$, which vanishes in the extreme limit. Finally, for the de Sitter geometry, we write

$$S = N \ln 2 = \frac{3\pi}{l_p^2 \Lambda_c} = \frac{A}{4l_p^2} = \frac{N4\pi l_p^2}{4l_p^2} = N\pi, \quad (14)$$

that is

$$\frac{3\pi}{l_p^2 N \ln 2} = \Lambda_c. \quad (15)$$

An interesting aspect appears when we put numbers in Eq.(15). When $N = 1$, the foam system is highly unstable and the cosmological constant assumes the value, in order of magnitude, of $\Lambda_c \sim 10^{38} GeV^2$. However the system becomes stable when the whole universe has been filled with wormholes of Planckian size and this leads to the huge number $N = 10^{122}$ corresponding to the value of $\Lambda_c \sim 10^{-84} GeV^2$ which is the order of magnitude of the cosmological constant of the space in which we now live.

2.2 Lorentz invariance?

Recently, a lot of interest has been devoted to the problem of Lorentz invariance at high energies. There are several reasons to suspect that Lorentz invariance may be only a low energy symmetry. This possibility is suggested by the ultraviolet divergences of local quantum field theory, as well as by tentative results in various approaches to quantum gravity and string theory [12]. One signal of breaking of such a symmetry should be given by a modification of the dispersion relation deviating from

$$E^2(p) = p^2 + m^2, \quad (16)$$

where for simplicity we have considered the case of a massive scalar field. One possible simple distortion can be characterized at low energies by an expansion with integral powers of momentum,

$$E^2 = m^2 + Ap^2 + Bp^3/E_0 + Cp^4/E_0^2 + O(p^5). \quad (17)$$

E_0 is the ‘‘quantum gravity’’ scale (Planck energy $\simeq 10^{19}$ GeV). The dispersion relation is not Lorentz invariant. It can only hold in one reference frame. One possible ‘‘preferred reference frame’’ could be represented by space-time foam³. In our foam model we have the possibility to verify the above dispersion relation, which can be also used to probe the validity of this picture.

³See also Ref.[13], where Lorentz symmetry has been explicitly broken.

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