



Traversable wormholes from massless conformally couple scalar fields

Carlos Barcelo
Washington University
carlos@robert.wustl.edu

ABSTRACT

The massless conformally coupled scalar field is characterized by the so-called "new improved stress-energy tensor", which is capable of classically violating the null energy condition. When coupled to Einstein gravity we find a three-parameter class of exact solutions. These exact solutions include the Schwarzschild geometry, assorted naked singularities, and a large class of traversable wormholes.

TRAVERSABLE WORMHOLES
FROM
MASSLESS CONFORMALLY COUPLED
SCALAR FIELDS

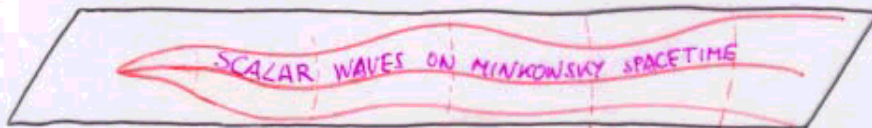
CARLOS BARCELO & MATT VISSER

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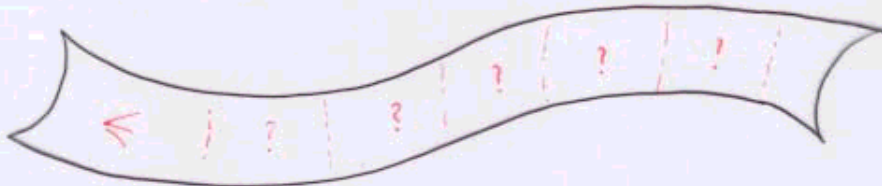
TO APPEAR IN PLB

SCALAR FIELDS ON CURVED SPACETIMES

SUPPOSE THAT WE KNOW THE BEHAVIOUR OF A SCALAR FIELD ON A MINKOWSKI SPACETIME



WHAT IS ITS BEHAVIOUR WHEN THE SPACETIME IS CURVED?



NO UNIQUE PRESCRIPTION!

- MINIMUM COUPLING

• MINIMAL SUBSTITUTION ($\partial_j \rightarrow \nabla_j$)

- CONFORMAL COUPLING

• A POTENTIAL TERM ADDED ($\mathcal{L}_c(\phi) = \mathcal{L}_M(\phi) - \frac{\sqrt{-g}}{12} R \phi^2$)

- OTHERS

• $\mathcal{L}_\xi(\phi) = \mathcal{L}_M(\phi) - \frac{\sqrt{-g}}{2} \xi R \phi^2$ $0 \neq \xi \neq \frac{1}{6}$

* WE WILL RESTRICT FOR SIMPLICITY TO A FREE MASSLESS SCALAR FIELD

CONFORMAL COUPLING

- KEEPS THE CONFORMAL INVARIANCE.
 - MIMICS THE PHOTONS BEHAVIOR.
- ITS ENERGY-MOMENTUM TENSOR YIELD THE "NEW IMPROVED ENERGY-MOMENTUM TENSOR" OF PARTICLE PHYSICS WHEN PARTICULARIZED TO FLAT SPACETIME.
 - FINITE MATRIX ELEMENTS IN RENORMALIZED PERTURBATION THEORY.
 - NATURAL BEHAVIOUR AT LOW ENERGIES

SCALAR FIELD AS THE SOURCE OF CURVATURE

MINIMAL
COUPLING



CONFORMAL
COUPLING

CLASSICAL SOLUTIONS
GRAVITY + MINIMALLY
COUPLED SCALAR



CLASSICAL SOLUTIONS
GRAVITY + CONFORMALLY
COUPLED SCALAR

POINT WISE AND AVERAGED NULL ENERGY CONDITIONS

EINSTEIN EQUATIONS

$$\kappa G_{\mu\nu} = T_{\mu\nu}(\phi, \dot{\phi}) \quad \kappa = \frac{1}{8\pi G}$$

LET $x^\mu(\lambda)$ BE A LIGHT LIKE CURVE WITH
TANGENT VECTOR $\kappa^\mu = \frac{dx^\mu}{d\lambda}$

$$\kappa^\mu \kappa^\nu T_{\mu\nu}(\phi) = (1 - \dot{\phi}^2/6\kappa)^{-1} \left[\frac{2}{3} \left(\frac{d\phi}{d\lambda} \right)^2 - \frac{1}{3} \phi \frac{d^2\phi}{d\lambda^2} \right]$$

CAN BE NEGATIVE!

ALSO $\int d\lambda \kappa^\mu \kappa^\nu T_{\mu\nu}(\phi)$ CAN BE NEGATIVE!

IT CAN BE SITUATIONS IN WHICH EVEN A NULL
GEODESIC OBSERVER WILL SEE
NEGATIVE ENERGY DENSITIES

SO...

CAN A CONFORMAL SCALAR FIELD SUPPORT
TRAVERSABLE WORMHOLE GEOMETRIES?

STATIC AND SPHERICALLY SYMMETRIC SOLUTIONS

EQUATIONS:

1) $R=0$

2) $\kappa G_{00} = T_{00}(\phi)$

3) $\square\phi=0$

MINIMALLY COUPLED SCALAR FIELD SOLUTIONS (JNW)

$$ds_H^2 = -\left(1 - \frac{2\gamma}{r}\right)^{\cos\chi} dt^2 + \left(1 - \frac{2\gamma}{r}\right)^{-\cos\chi} dr^2 + \left(1 - \frac{2\gamma}{r}\right)^{1-\cos\chi} r^2 d\Omega_2^2$$

$$\phi_H = \sqrt{\frac{\kappa}{2}} \sin\chi \ln\left(1 - \frac{2\gamma}{r}\right)$$

LET NOW $ds_c^2 = \Omega^2(r) ds_H^2$

EQUATIONS 1, 2, 3

$$\Omega = \alpha_+ \exp(+\phi_H/\sqrt{6\kappa}) + \alpha_- \exp(-\phi_H/\sqrt{6\kappa})$$

$$\phi_c = \pm\sqrt{6\kappa} \left[\frac{\alpha_+ \exp(+\phi_H/\sqrt{6\kappa}) - \alpha_- \exp(-\phi_H/\sqrt{6\kappa})}{\alpha_+ \exp(+\phi_H/\sqrt{6\kappa}) + \alpha_- \exp(-\phi_H/\sqrt{6\kappa})} \right]$$

GENERALIZATION OF THE SOLUTIONS

}	FROYLAND
	AGNESE & LA CAMERA

PARAMETER SPACE

DEFINITION: $\tan(\Delta/2) = \frac{\alpha_+ - \alpha_-}{\alpha_+ + \alpha_-}$

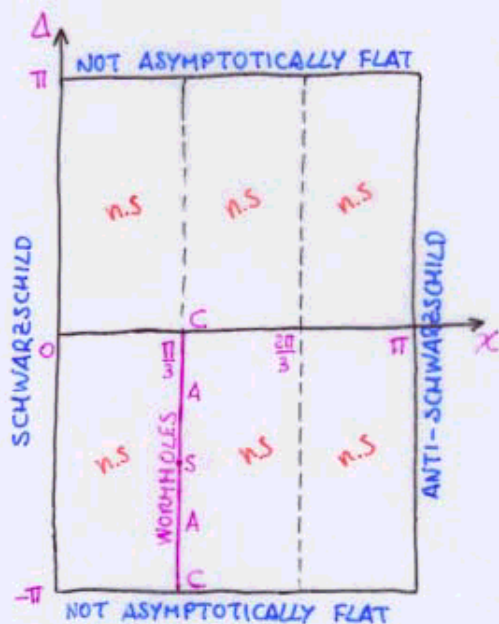
- THREE PARAMETER FAMILY OF SOLUTIONS

z ; $\chi \in (-\pi, \pi]$; $\Delta \in (-\pi, \pi)$

RELATED WITH : M, Q, ϕ_a

- AFTER SOME IDENTIFICATIONS $\left\{ \begin{array}{l} (z, \chi, \Delta) \cong (z, -\chi, -\Delta) \\ (z, \chi, \Delta) \cong (-z, \chi + \pi, \Delta) \end{array} \right\}$

WE ONLY HAVE TO DEAL WITH $z \geq 0, \chi \in [0, \pi], \Delta \in (-\pi, \pi]$



TRAVERSABLE WORMHOLES GEOMETRIES

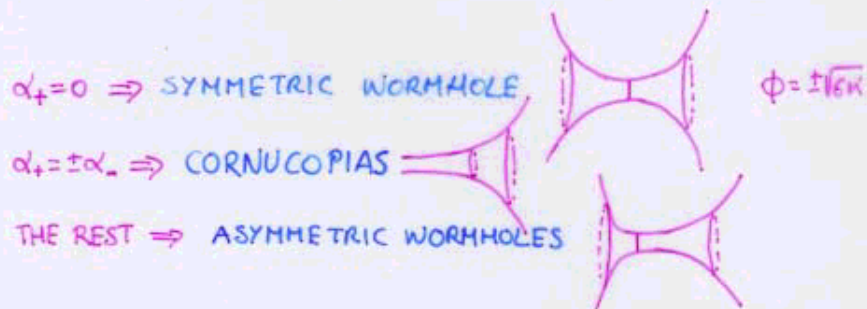
BY USING ISOTROPIC COORDINATES $r = \bar{r} \left(1 + \frac{\gamma}{2\bar{r}}\right)^2$

WE CAN WRITE:

$$ds_c^2 = \underbrace{\left[\alpha_+ \frac{\left(1 - \frac{\gamma}{2\bar{r}}\right)}{\left(1 + \frac{\gamma}{2\bar{r}}\right)} + \alpha_- \right]}_{\Omega'(\bar{r})} \left[-dt^2 + \left(1 + \frac{\gamma}{2\bar{r}}\right)^4 (d\bar{r}^2 + \bar{r}^2 d\Omega_2^2) \right]$$

FOR $\frac{\alpha_+ - \alpha_-}{\alpha_+ + \alpha_-} < 0 \Rightarrow$ WORMHOLE GEOMETRIES : $\Omega'(\bar{r}) \neq 0 \forall \bar{r} \in [0, +\infty)$

$$\phi = \pm \sqrt{6\kappa} \frac{\alpha_+ \left(1 - \frac{\gamma}{2\bar{r}}\right) - \alpha_- \left(1 + \frac{\gamma}{2\bar{r}}\right)}{\alpha_+ \left(1 - \frac{\gamma}{2\bar{r}}\right) + \alpha_- \left(1 + \frac{\gamma}{2\bar{r}}\right)}$$

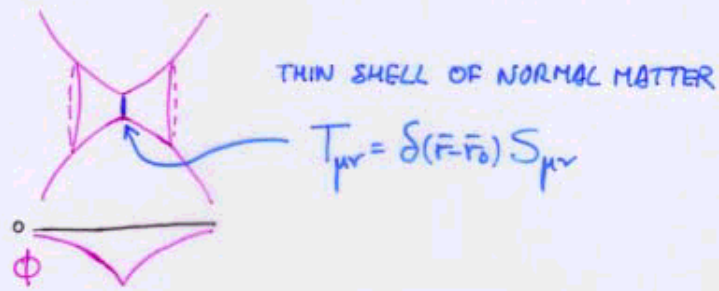


IN ALL THESE WORMHOLES $G_{\text{eff}} = G \left(1 - \frac{\phi^2}{6\kappa}\right)^{-1}$

HAS A DIFFERENT SIGN ON EACH ASYMPTOTIC REGION

WORK IN PROGRESS

- BUILDING "SYMMETRIC" WORMHOLES



- GENERALIZATION TO ARBITRARY ξ COUPLINGS

- HOW THE VALUE $\xi = \frac{1}{4}$, THRESHOLD FOR ANEC VIOLATIONS, SHOWS UP ON THESE CONSTRUCTIONS?