Wormholes and Time Travel? Not Likely

L. Susskind

Department of Physics
Stanford University
Stanford, CA 94305-4060

Abstract

Wormholes have been advanced as both a method for circumventing the limitations of the speed of light as well as a means for building a time machine (to travel to the past). Thus it is argued that General Relativity may allow both of these possibilities. In this note I argue that traversable wormholes connecting otherwise causally disconnected regions, violate two of the most fundamental principles physics, namely local energy conservation and the energy-time uncertainty principle.

1 Wormholes

A wormhole is a multiply connected spatial geometry that consists of two "mouth holes" separated by some distance. The lips of the mouths are identified so that a shortcut exists through the wormhole. Going the long way, the distance between the mouths, call it L, may be huge or even infinite. By contrast the distance l, through the wormhole, may be very small. According to some authors, such wormholes might be left over from the big bang, or it might even be possible to construct them. They are very common, being frequently seen in the press, in science fiction stories, and in physics books for a popular audience.

Such wormholes, if they exist, would be very interesting windows on distant regions of space–even regions beyond our cosmic horizon. We might pass through one, take a look around, and come back to report what we saw [1][2]. If we were not bold enough to do that, we could send a TV camera connected to a cable. Wormholes have even been advocated as a means to escape a dying bubble-universe to a younger safer one [3]. If they existed they could give operational meaning to the "Multiverse" idea. Finally, they are purported components of time machines [4].

In analyzing these claims, it is important to realize that traversable wormholes (TW's) are not possible in classical physics. Classical GR does not allow a wormhole such as an Einstein Rosen Bridge, to stay open long enough for an object to cross from one mouth to the other. The idea is that quantum mechanics, usually in the form of the Casimir effect, can allow violations of the positive energy conditions that prohibit TW's. I am going to give wormholes the benefit of the doubt and assume that the quantum effects can stabilize them. However, because they do not exist in the classical limit, a consistent analysis must take into account the constraints of quantum mechanics, in particular the uncertainty principle. That, and the fact that energy is a gauge charge, is all will use in this paper.

Purely for the purposes of being able to visualize the geometry, I am going to analyze the question in 2+1 dimensions. Let us begin by describing a two dimensional spatial geometry containing a wormhole. We take the usual large sheet of paper and bend it so that distant points are close in three dimensional embedding space. Then we cut two mouths and identify the edges. If the long way around is very big we can ignore the fact that the two sheets are connected (except by the wormhole) and think of them as two separate universes connected by a wormhole. Call the two sheets **A** and **B**. We live on **A**.

Seen from either side, the wormholes may have mass. If so they create a gravitational field which in 2+1 dimensions means a conical deficit. In that case the geometry would be a pair of cones connected at their tips by a smoothed region of negative curvature. Incidently, the fact that the wormhole is negatively curved is an explicit demonstration that negative energy is needed to support the configuration.

2   The Electric Case

Let's begin with a simple experiment involving two boxes (boxes $\mathbf{1}, \mathbf{2}$) that can contain electrically charged particles. Call the particles π^+ mesons. We can construct a state (we work in Coulomb gauge) with total charge N and relative phase θ.

$$|N\theta\rangle = \sum_{n=0}^{N} |N - n, n\rangle e^{in\theta} \tag{2.1}$$

where $|m, n\rangle$ means a state with charge m, and n in box $\mathbf{1}$ and $\mathbf{2}$. The relative phase is a measurable [5] and can be measured as follows. A beam of neutrons is split into two beams. One beam is sent through box $\mathbf{1}$ and the other through box $\mathbf{2}$. In passing through the box of π^+ mesons, a charge may be absorbed, turning a neutron into a proton. When the beams are recombined, the probability to find a proton contains an interference term proportional to $\cos(2\theta)$.

Typically the phase will not be time independent but, because of the electrostatic coulomb energy, will oscillate in a regular predictable way. The boxes can be separated with the phase relations remaining intact.

Imagine the two boxes starting out on our side of the wormhole on sheet $\mathbf{A}$. Now transport box $\mathbf{2}$ to the other end of the wormhole, taking it the long way around. At the end of this process we have two charge reservoirs, one on each sheet, with definite relative phase. this provides us with a means of comparing phases at the two mouths.

Next, pass box $\mathbf{1}$ through the wormhole so that it appears on sheet $\mathbf{B}$. Now both boxes are on the same sheet and presumably have a definite phase that can be compared. But in fact when we experimentally compare the phases we find them to be completely uncorrelated: the interference term proportional to $\cos(2\theta)$ is absent.

To see why this is so, lets examine more closely what happens when a charge is passed through a wormhole. When the charge began on sheet $\mathbf{A}$, its electric flux lines radiated out to infinity on that side. As we pass the charge through the wormhole, its flux lines do not suddenly asymptotically rearrange. They thread back through the wormhole and

2

continue to radiate to infinity on sheet **A**. In other words NO CHARGE passed through the wormhole. In fact what the observer on side **B** sees is that in addition to the charge that passed through, an opposite charge has formed on the wormhole mouth. On side **A** the charge is also unchanged but it now exists in the form of a charged wormhole mouth.

Now lets return to the problem of two charged boxes with definite relative phases. After sending box **1** through the wormhole, there are four charged systems to keep track of–the two boxes and the two wormhole mouths, **A** and **B**. Instead of finding the phases of the two boxes correlated, what one finds is a phase correlation between box **2** and mouth **A**, and a similar correlation between the phases of **1** and **B**, but no correlation between phases **1** and **2**.

3   Time and Energy Transfer

Energy and time are related in the same way as charge and phase. Let's repeat the analysis and see what it says for this case. For ease of visualizing the system, we work in 2+1 dimensions but the analysis applies in higher dimensions.

First consider what happens when a mass passes through the wormhole from **A** to **B**. For simplicity consider a point mass. The analogue of the lines of electric flux is the conical deficit that extends to infinity on sheet **A**. As in the electric case, when the mass passes through the wormhole, the conical deficit remains behind and no change takes place in the deficit on either side. On side **A** the deficit appears to be due to mouth **A**. On side **B** the deficit is shared between the mass point and mouth **B**, which appears to have lost whatever mass is carried by the mass point that it spit out.

Let's start over. Instead of two boxes of charge that serve as fiducial phases, we take a given energy and partition it into two clocks, call them **1** and **2**. The phase variables are replaced by the time readings t_1, t_2. The clocks can be initially prepared so that the time difference, $t_1 - t_2$ is zero: in other words the clocks are synchronized.

Next transport clock **2** the long way to sheet **B**. If this is done with no great velocity and without passing through gravitational potential wells, the two clocks will remain synchronized. In any case there is no difficulty in accounting for any difference in proper times along the respective paths of the clocks. We now have clocks that can compared near the two mouths.

Here is the experiment that we would like to do. Pass clock **1** through the wormhole and compare it with clock **2**. After recording the result, clock **1** may return through the

wormhole and report its finding. One would naively expect that apart from effects that might occur in the wormhole, the two clocks should agree. In other words the relative time observable $(t_B - t_A)$ would be expected to be very small compared to L. But it is clear that this cannot be so. Instead the two clocks will be completely uncorrelated. This is a consequence of the energy-time uncertainty principle and the fact that the energy that passed through the wormhole was exactly zero. Thus we can conclude that

$$\langle (t_B - t_A)^2 \rangle = \infty. \tag{3.1}$$

Note that clock **1** is equally likely to come out in the infinitely remote past as in the infinitely remote future!

Another way to say what happened is that when clock **1** entered and exited at **A** and **B**, it left behind clocks at the wormhole mouths. After exiting at **B**, clock **1** found itself synchronized with the clock at mouth **B**, but totally random with respect to clock **2**.

4   Conclusion

Note added:

The conclusion described below has been seriously criticized in a recent rebuttal. See gr-qc/0504039.

Traversable wormholes that would allow an observer to short-circuit large space-like distances by entering one mouth and exiting another violate quantum mechanics. The average magnitude of the time between entering and exiting is infinite as a consequence of the uncertainty principle and the fact that the total ADM energy transfer must be exactly zero.

The only sensible interpretation is that the events at the two wormhole mouths are completely uncorrelated. The appearance of a clock similar to clock **1** on sheet **B** can be interpreted as a quantum emission by an unstable object, namely the mouth **B**. That decay can take place at any time and is uncorrelated to the "absorption" event at **A**. I think it is even justified to say that the observer never left **A**. It was left behind in the form of the complex energy levels of mouth **A**.

As for time travellers, they will need to find another kind of time machine.

5   Note added

Serguei Krasnikov has reminded me that there is an additional parameter needed to specify a wormhole, namely a relative time variable that specifies how the identification is done. For example two stationary wormholes mat be identified at equal time in the rest frame or there may be an offset by some finite delay or advance. Obviously if the delay is finite this will make no difference to the argument, even if the delay time is somewhat uncertain, quantum mechanically.

One way of fixing the time delay is to start with a wormhole that is short both the "long way" and through the wormhole. Some authors have speculated that such small wormholes might be created in the lab. Once created, they can be slowly separated to a large distance. If separated symmetrically, the delay time must be zero by symmetry. Moreover, there is no reason why the delay would fluctuate beyond whatever small fluctuation may have been there in the beginning.

References

[1] Kip Thorne, Black Holes and Time Warps

[2] Brian Greene, The Fabric of the Cosmos: Space, Time, and the Texture of Reality

[3] M. Kaku, Parallel Worlds

[4] Other kinds of classical time machines have been convincingly critisized in S. Deser, R. Jackiw, G. 't Hooft, PRL 68, 267(1992), See also S. Deser, Comments in Nucl Part Phys 20,337 (1992)

[5] Y. Aharonov and L. Susskind, Charge Superselection Rule Phys. Rev. 155, 1428 (1967)