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Can wormholes exist?

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Abstract

Renormalized vacuum expectation values of electromagnetic stress-energy tensor are calculated in the background spherically-symmetrical metric of the wormhole's topology. Covariant geodesic point separation method of regularization is used. Violation of the weak energy condition at the throat of wormhole takes place for geometry sufficiently close to that of infinitely long wormhole of constant radius irrespectively of the detailed form of metric. This is an argument in favour of possibility of existence of selfconsistent wormhole in empty space maintained by vacuum field fluctuations in the wormhole's background.

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1.Introduction. In 1988 Kip Thorne with co-workers [1],[2] have studied properties of static spherically-symmetrical traversable wormhole. To exist, such the wormhole should be threaded by material with rather unusual properties. In particular, radial pressure of the material should exceed it's density both locally at the throat [1] and integrally along radial direction [2]. That is, weak energy condition (WEC) [3] at the throat and averaged weak energy condition (AWEC) [4] should be violated. Where can we find such the material? In [2] Casimir vacuum between conducting spherical plates surrounding the throat was considered as a kind of such the material since it possesses required properties. In the given note another possibility is studied: self-maintained wormhole threaded by electromagnetic field vacuum. Covariantly renormalized vacuum expectation values (VEV's) of electromagnetic stress-energy tensor are found to violate WEC at the throat of the wormhole if it's typical length (proper radial distance at which variation of radius becomes comparable to the radius itself) is large as compared with it's radius. As for AWEC, check of it requires more complicated calculation and will be given in forthcoming paper.

2.Calculation. In calculation it is convenient to use instead of radius r the radial distance ρ, so interval will take the form

$$ds^2 = r^2(\rho)d\Omega^2 + d\rho^2 - \exp{(2\Phi)}dt^2, \quad d\Omega^2 = d\theta^2 + \sin^2\theta\, d\phi^2 \tag{1}$$

In the case of wormhole Φ is everywhere finite [1]. Consider electromagnetic field in this metric background, $F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$, in the gauge $A_t = 0$. There exist transversal electric TE and transversal magnetic TM modes with components of vector potential of the form

$$\mathrm{TE} : (A_\rho, A_\theta, A_\phi) = (0,\ \frac{Y_\phi}{\sin\theta}\ ,\ - Y_\theta\ sin\theta)R \tag{2}$$

$$\mathrm{TM} : (A_\rho, A_\theta, A_\phi) = (l(l+1)\frac{R}{r^2}Y,\ R_\rho Y_\theta,\ R_\rho Y_\phi)\frac{\exp{(\Phi)}}{\omega} \tag{3}$$

Here $Y \equiv Y_{lm}(\theta,\phi)$, $R \equiv R_{nl}(\rho)$, $\omega \equiv \omega_{nl}$, $Y_\theta \equiv \partial Y/\partial\theta$, $Y_\phi \equiv \partial Y/\partial\phi$, $R_\rho \equiv dR/d\rho$. Radial functions obey eigenvalue equation

$$-\left(\exp{(\Phi)}\frac{d}{d\rho}\right)^2 R + l(l+1)\frac{\exp{(2\Phi)}}{r^2}R = \omega^2 R \tag{4}$$

with boundary conditions on conducting spherical plates Γ (if any)

$$\mathrm{TE} :\ R|_\Gamma = 0; \quad \mathrm{TM} :\ R_\rho|_\Gamma = 0. \tag{5}$$

It is apparently conformally invariant.

Substitute appropriately normalized solutions for it into split-regularized form of stress-energy tensor

$$T^{\mu\nu}(x) = \lim_{\tilde{x}\to x} [g^{\mu\lambda}(x)g^{\nu\omega}(x) - \frac{1}{4}g^{\mu\nu}(x)g^{\lambda\omega}(x)]\frac{1}{2}g^{\varrho\tau}(x)[F_{\lambda\varrho}(x)F_{\omega\tau}(\tilde{x})_x + F_{\lambda\varrho}(\tilde{x})_x F_{\omega\tau}(x)] \tag{6}$$

and sum over eigenmodes (over $n,\ l,\ m$ and polarization $\sigma =$ TE or TM). Here notation $F_{\mu\nu}(\tilde{x})_x$ means components of tensor $F_{\mu\nu}(\tilde{x})$ transported in parallel way from point $\tilde{x}$ to x along the geodesic connecting both points. It is convenient to split the point only in coordinate ρ; then parallel transport defined by vanishing the covariant derivative means simply constancy of "physical" components,

$$A_{\hat{\mu}\hat{\nu}\ldots\hat{\lambda}} \stackrel{\mathrm{def}}{=} |g_{\mu\mu}g_{\nu\nu}\ldots g_{\lambda\lambda}|^{-1/2}A_{\mu\nu\ldots\ lambda} = \mathrm{const}, \tag{7}$$

of any transported in such the way tensor A. Notations coincide with those of ref.[1] and are convenient in what follows. Normalization of radial functions should enable contribution of each eigenmode to energy

$$-\int T^{\mu}{}_{t}g^{1/2}dS_{\mu} = -\int T^{t}{}_{t}g^{1/2}d\rho d\theta d\phi \quad (g \equiv -\det\|g_{\mu\nu}\|) \tag{8}$$

to take on vacuum value $\hbar\omega/2$ (this is just *covariant* component of vector, as it should be for energy quantum mechanically conjugated to *contravariant* time). By properties of spherical functions summation over m eliminates angle dependence. The resulting vacuum expectation values of components of stress-energy tensor will be denoted just as components themselves; this will not lead to any confusion. So we have for nonzero components

$$\begin{aligned}
T_{\hat{t}\hat{t}} &= \lim_{\epsilon\to 0}\sum_{n,l,\mathrm{TE},\mathrm{TM}} \frac{l+1/2}{8\pi\omega r\tilde{r}}\frac{R_\rho\tilde{R}_\rho + \omega^2 R\tilde{R}\exp(-\Phi-\Phi') + R\tilde{R}\frac{l(l+1)}{r\tilde{r}}}{\int R^2\exp(-\Phi)d\rho}\\
T_{\hat{\rho}\hat{\rho}} &= \lim_{\epsilon\to 0}\sum_{n,l,\mathrm{TE},\mathrm{TM}} \frac{l+1/2}{8\pi\omega r\tilde{r}}\frac{R_\rho\tilde{R}_\rho + \omega^2 R\tilde{R}\exp(-\Phi-\Phi') - R\tilde{R}\frac{l(l+1)}{r\tilde{r}}}{\int R^2\exp(-\Phi)d\rho}\\
T_{\hat{\theta}\hat{\theta}} &= \lim_{\epsilon\to 0}\sum_{n,l,\mathrm{TE},\mathrm{TM}} \frac{l+1/2}{8\pi\omega r\tilde{r}}\frac{R\tilde{R}\frac{l(l+1)}{r\tilde{r}}}{\int R^2\exp(-\Phi)d\rho}
\end{aligned} \tag{9}$$

Here $\epsilon = |\tilde{\rho}-\rho|$, $\tilde{R} \equiv R(\tilde{\rho})$, $\tilde{r} \equiv r(\tilde{\rho})$ and hereafter the components of any tensor obtained by interchange $\hat{\theta} \leftrightarrow \hat{\phi}$ are the same and will not be written out.

Due to formal conformal invariance of electromagnetic field it is natural to temporarily change variable $\rho \to z$ via $dz = \exp(-\Phi)d\rho$ and to consider the sums

$$\begin{aligned}
\sum_l (l+\frac{1}{2})\sum_n \frac{R_z\tilde{R}_z}{\omega} &= S_{\mathrm{r}}\ (\equiv S_{\mathrm{radial}})\\
\sum_l (l+\frac{1}{2})\sum_n \omega R\tilde{R} &= S_{\mathrm{f}}\ (\equiv S_{\mathrm{full}})
\end{aligned}$$

$$\begin{aligned}\sum_l (l+\frac{1}{2})l(l+1)\sum_n \frac{R\tilde{R}}{\omega} &= S_{\rm a}\ (\equiv S_{\rm angle}) \\ \sum_l (l+\frac{1}{2})\sum_n \frac{R\tilde{R}}{\omega} &= U\end{aligned} \tag{10}$$

Then $S_{\rm r} = U_{z\tilde{z}}$ and, by eq.(4) , $S_{\rm f} = -U_{z^2} + r^{-2}\exp{(2\Phi)}S_{\rm a}$ so that

$$\begin{aligned}8\pi T_{\hat{t}\hat{t}} &= S_{\rm a}\frac{\exp{(-\Phi)}}{r^2\tilde{r}}\left(\frac{\exp\Phi}{r} + \frac{\exp{(\tilde{\Phi})}}{\tilde{r}}\right) + \frac{\exp{(-\Phi-\tilde{\Phi})}}{r\tilde{r}}U_{z(\tilde{z}-z)} \\ 8\pi T_{\hat{\rho}\hat{\rho}} &= S_{\rm a}\frac{\exp{(-\Phi)}}{r^2\tilde{r}}\left(\frac{\exp{(\Phi)}}{r} - \frac{\exp{(\tilde{\Phi})}}{\tilde{r}}\right) + \frac{\exp{(-\Phi-\tilde{\Phi})}}{r\tilde{r}}U_{z(\tilde{z}-z)} \\ 8\pi T_{\hat{\theta}\hat{\theta}} &= S_{\rm a}\frac{1}{r^2\tilde{r}^2}\end{aligned} \tag{11}$$

Computation of sums over n in the above expressions reduces to that of a Green function by the following trick. We introduce

$$G^{\rm E\ or\ M} \equiv G_l^{\rm E\ or\ M}(t,\rho,\tilde{\rho}) = \sum_{n,{\rm TE\ or\ TM}} \frac{R\tilde{R}}{t^2+\omega^2}, \quad G = G^{\rm E} + G^{\rm M} \tag{12}$$

through which U and $S_{\rm a}$ are expressible:

$$\begin{aligned}U &= \sum_{l=1}^{\infty}\int_0^{\infty}\frac{2l+1}{\pi}G dt \equiv \sum\!\!\!\!\!\!\int \frac{2l+1}{\pi}G dt \\ S_{\rm a} &= \sum\!\!\!\!\!\!\int \frac{2l+1}{\pi}l(l+1)G dt\end{aligned} \tag{13}$$

In turn, $G^{\rm E},\ G^{\rm M}$ are some Green functions:

$$-G_{zz}^{\rm E,M} + \left(t^2 + l(l+1)\frac{\exp{(2\Phi)}}{r^2}\right)G^{\rm E,M} = \delta(z-z') \tag{14}$$
$$G^{\rm E}|_{z\in\Gamma} = 0, \quad G_z^{\rm M}|_{z\in\Gamma} = 0$$

These eqs. are then solved perturbatively. Put

$$\frac{\exp{(2\Phi)}}{r^2} = \frac{\exp{(2\Phi_0)}}{r_0^2} + V, \tag{15}$$

where index 0 denote values of considered quantities at the throat (at $\rho = 0$), and expand over V.

We are interested here in empty space and thus shift Γ to infinity; then it proves that $G^{\rm E} = G^{\rm M} \equiv \bar{G},\ G = 2\bar{G}$. In zero approximation ($V = 0$)

$$\bar{G}^{(0)} = \frac{1}{2m} \exp\left(-m|z-\tilde{z}|\right), \quad m^2 \equiv t^2 + l(l+1)\frac{\exp(2\Phi_0)}{r_0^2}. \tag{16}$$

In the first (linear in V) approximation

$$G = 2\bar{G}^{(0)}(z,\tilde{z}) - 2l(l+1)\int \bar{G}^{(0)}(z,y)V(y)\bar{G}^{(0)}(y,\tilde{z})dy \tag{17}$$

Calculating so, we are faced with integrosums over $l,\ dt$. There are two basic ones, and these arise already in zero approximation. In this approximation, substituting variables $t = \exp(\Phi_0) r_0^{-1}[l(l+1)]^{1/2}\sinh\varphi$ and then $\epsilon\cosh\varphi = r_0 q$ we get with the help of (16), (13) and (11)

$$2\pi^2 r_0^4 \begin{pmatrix} T_{\hat{t}\hat{t}} \\ T_{\hat{\rho}\hat{\rho}} \\ T_{\hat{\theta}\hat{\theta}} \end{pmatrix} = \begin{pmatrix} -I_1(\epsilon/r_0) \\ -I_1(\epsilon/r_0) - I_2(\epsilon/r_0) \\ I_2(\epsilon/r_0)/2 \end{pmatrix} \tag{18}$$

where

$$I_1(\epsilon) = \frac{1}{\epsilon^2}\int_\epsilon^\infty \frac{(q^2-\epsilon^2)^{1/2}}{q^4} h(q)dq, \quad I_2(\epsilon) = \int_\epsilon^\infty \frac{h(q)dq}{q^4(q^2-\epsilon^2)^{1/2}} \tag{19}$$

and function $h(q)$ is regular at $q=0$. Carefully expanding it under the integral sign in Taylor series we get

$$\begin{aligned}
I_1(\epsilon) &= \frac{1}{3}\frac{h(0)}{\epsilon^4} + \frac{\pi}{4}\frac{h'(0)}{\epsilon^3} + \frac{1}{2\epsilon^2}\left[\int_0^\infty \frac{h''(q) - h''(0)\theta(M-q)}{q}dq + h''(0)\left(\ln\frac{2M}{\epsilon} + \frac{1}{2}\right)\right] \\
&\quad - \frac{\pi}{12}\frac{h'''(0)}{\epsilon} - \frac{1}{48}\left[\int_0^\infty \frac{h^{\text{iv}}(q) - h^{\text{iv}}(0)\theta(M-q)}{q}dq + h^{\text{iv}}(0)\left(\ln\frac{2M}{\epsilon} + \frac{31}{12}\right)\right], \\
I_2(\epsilon) &= \frac{2}{3}\frac{h(0)}{\epsilon^4} + \frac{\pi}{4}\frac{h'(0)}{\epsilon^3} + \frac{1}{2}\frac{h''(0)}{\epsilon^2} + \frac{\pi}{12}\frac{h'''(0)}{\epsilon} \\
&\quad + \frac{1}{24}\left[\int_0^\infty \frac{h^{\text{iv}}(q) - h^{\text{iv}}(0)\theta(M-q)}{q}dq + h^{\text{iv}}(0)\left(\ln\frac{2M}{\epsilon} + \frac{25}{12}\right)\right]
\end{aligned} \tag{20}$$

For $h(q)$ we have

$$\begin{aligned}
h(q) &= q^4 \sum_{l=1}^\infty \left(l + \frac{1}{2}\right) l(l+1) \exp\{-q[l(l+1)]^{1/2}\} \\
&= q^4 \frac{d^2}{dq^2}\frac{f(q)}{q^2} = q^2 f''(q) - 4qf'(q) + 6f(q), \\
f(q) &= q^2 \sum_{l=1}^\infty \left(l + \frac{1}{2}\right) \exp\{-q[l(l+1)]^{1/2}\}
\end{aligned} \tag{21}$$

The $f(q)$ is also regular at $q=0$. Thus $h(0) = 6f(0)$, $h'(0) = 2f'(0)$, $h''(q) = q^2 f^{\text{iv}}(q)$, $h'''(0) = 0$, $h^{\text{iv}}(0) = 2f^{\text{iv}}(0)$. In $f(q)$ we expand exponential denoting $l + \frac{1}{2} = \gamma$:

$$\exp\left[-q\left(\gamma^2-\frac{1}{4}\right)^{1/2}\right]=\exp\left(-q\gamma\right)\left[1+qF+\frac{1}{2}q^2F^2+O(q^3F^3)\right],$$

$$F=\gamma\left[1-\left(1-\frac{1}{4\gamma^2}\right)^{1/2}\right]=\frac{1}{8\gamma}+\frac{1}{128\gamma^3}+O(\gamma^{-5}) \tag{22}$$

This is enough to express required derivatives of $f(q)$ at zero in terms of those of elementary sum $g(q)=q\sum\exp\left(-q\gamma\right)$. Lengthy also elementary calculation ultimately gives

$$2\pi^2r_0^4\begin{pmatrix}T_{\hat{t}\hat{t}}\\ T_{\hat{\rho}\hat{\rho}}\\ T_{\hat{\theta}\hat{\theta}}\end{pmatrix}^{(0)}_{\mathrm{reg}}=\begin{pmatrix}-2\frac{r_0^4}{\epsilon^4}+\frac{1}{3}\frac{r_0^2}{\epsilon^2}-\frac{1}{60}\ln\frac{L}{\epsilon}\\ -6\frac{r_0^4}{\epsilon^4}+\frac{1}{3}\frac{r_0^2}{\epsilon^2}+\frac{1}{60}(\ln\frac{L}{\epsilon}-1)\\ +2\frac{r_0^4}{\epsilon^4}\qquad\quad-\frac{1}{60}(\ln\frac{L}{\epsilon}-\frac{1}{2})\end{pmatrix} \tag{23}$$

for regularized stress-energy tensor in zero approximation (for space-time taken as direct product of sphere and Minkowsky plain). The nonlogarithmic $O(h^{\mathrm{iv}})$ contribution in $I_1,\ I_2$ cannot be accurately estimated within our elementary approach and is simply included into logarithm thus resulting in the appearance of effective cut off $L=kr_0$ with some numerical value k of the order of unity.

Now one renormalizes gravity action by adding to it (and subtracting from matter action) some functional W_{div} so that

$$T^{\mu\nu}_{\mathrm{div}}=g^{-1/2}\frac{\delta W_{\mathrm{div}}}{\delta g_{\mu\nu}} \tag{24}$$

cancels divergences in matter $T^{\mu\nu}$ [5]. This results in renormalization of cosmological constant, Einstein gravity constant and Weyl tensor invariant term in the action,

$$\int C_{\mu\nu\lambda\omega}C^{\mu\nu\lambda\omega}g^{1/2}d^4x, \tag{25}$$

where $C_{\mu\nu\lambda\omega}$ is conformal Weyl tensor. The full expression for $T^{\mu\nu}_{\mathrm{div}}$ was given by Christensen [6]. Nontrivial here is appearance of finite part of $T^{\mu\nu}_{\mathrm{div}}$ leading to nonzero trace of renormalized stress-energy tensor,

$$T^{\mu\nu}_{\mathrm{ren}}=T^{\mu\nu}_{\mathrm{reg}}-T^{\mu\nu}_{\mathrm{div}}, \tag{26}$$

so that

$$T^{\mu}_{\ \mu,\mathrm{ren}}=\frac{1}{2880\pi^2}\left(-13R_{\mu\nu\lambda\omega}R^{\mu\nu\lambda\omega}+88R_{\mu\nu}R^{\mu\nu}-25R^2-18\Box R\right) \tag{27}$$

Christensen's result depends locally on Riemannian and metric tensors and on ϵ^μ, the vector tangential at x to geodesic connecting points x and x', with length ϵ, the length of geodesic. In our case $\epsilon^\mu=(0,\ 0,\ \epsilon,\ 0)$. In zero approximation (space-time=sphere$\times$plain) the Riemannian tensor has the only (up to index permutations) nonzero component

$R_{\hat{\theta}\hat{\phi}\hat{\theta}\hat{\phi}} = r_0^{-2}$. Substitute these input values into Christensen's formula and subtract the resulting $T^{\mu\nu}_{\rm div}$ from our answer (23) for $T^{\mu\nu}_{\rm reg}$. All the divergent terms,ϵ^{-4}, ϵ^{-2} and $\ln\epsilon$ ones thus get cancelled. On the contrary, their cancellation is a useful check of our calculation. In logarithms the UV regulator ϵ is substituted by infrared one Λ. The finite part of $T^{\mu\nu}_{\rm div}$ turns out to be nonzero in this simple case of geometry only for one, $\rho\rho$-component:

$$2\pi^2 r_0^4 \begin{pmatrix} T_{\hat{t}\hat{t}} \\ T_{\hat{\rho}\hat{\rho}} \\ T_{\hat{\theta}\hat{\theta}} \end{pmatrix}^{(0)}_{\rm div,finite} = \begin{pmatrix} 0 \\ -\frac{1}{60} \\ 0 \end{pmatrix}, \tag{28}$$

i.e. it is completely consists of anomaly. The resulting renormalized tensor takes the form

$$2\pi^2 r_0^4 \begin{pmatrix} T_{\hat{t}\hat{t}} \\ T_{\hat{\rho}\hat{\rho}} \\ T_{\hat{\theta}\hat{\theta}} \end{pmatrix}^{(0)}_{\rm ren} = \begin{pmatrix} \frac{1}{60}\ln\frac{\Lambda}{L} \\ -\frac{1}{60}\ln\frac{\Lambda}{L} \\ \frac{1}{60}\ln\frac{\Lambda}{L} + \frac{1}{120} \end{pmatrix}. \tag{29}$$

The $\rho - t$ symmetry inherent in this simple case is restored just due to anomaly.

We now consider deviation of Φ, r from constants. Since we cannot take into account such deviation precisely, we need some parameter to expand over it. This parameter may be the ratio of wormhole's radius $r(0)$ to the typical length - the proper radial distance ρ at which the variation $r(\rho) - r(0)$ is comparable to $r(0)$. In practice, it is equivalent to expansion over the full number of differentiations $\frac{d}{d\rho}$. For example, Φ'' and Φ'^2 are considered in such the expansion on equal footing. At the throat odd derivatives vanish due to symmetry, and the nearest correction to zero approximation will be given by Φ''-, r''-terms arising in linear approximation over deviation of Φ, r from constants. This will lead to the following three types of contributions to $T^{\mu\nu}_{\rm reg}$ at the throat ($\rho = 0$).

First, those due to dependence on ϵ of metric factors in (11) taken at $\tilde{x}$; these follow by expanding in Taylor series in ϵ. At the throat we have:

$$\begin{aligned} \exp(\tilde{\Phi}) &= \exp(\Phi_0)\left[1 + \frac{1}{2}\Phi_0''\epsilon^2 + O(\epsilon^4)\right], \\ \tilde{r} &= r_0 + \frac{1}{2}r_0''\epsilon^2 + O(\epsilon^4) \end{aligned} \tag{30}$$

(here $r_0'' \equiv r''(0)$ etc.).

Second, those connected with changing expression of $|\tilde{z} - z| = |\int_0^\epsilon \exp(-\Phi)d\rho|$ entering already found formulas for $T^{\mu\nu\,(0)}_{\rm reg}$ in terms of geodesic length ϵ; to take this changing into account one should substitute in these formulas ϵ by

$$\epsilon\left[1 - \frac{1}{6}\Phi_0''\epsilon^2 + O(\epsilon^4)\right]$$

Third, contribution coming from linear in V correction to Green function G. Here we simply substitute Taylor series for V around $y = 0$ into perturbative expansion (17) for G and integrate over y; due to exponents integration for each Taylor term is finite. Appearing integrosums reduce to that of above considered type I_2, but now $q^{-4}h$ is less singular at $q = 0$ and/or it gets multiplied by positive power of ϵ.

These contributions are collected in the resulting total correction $T^{\mu\nu\,(1)}_{\rm reg}$, respectively:

$$
2\pi^2 \begin{pmatrix} T_{\hat t\hat t} \\ T_{\hat\rho\hat\rho} \\ T_{\hat\theta\hat\theta} \end{pmatrix}^{(1)}_{\rm reg} = \begin{pmatrix} 2\frac{\Phi_0''}{\epsilon^2} - \frac{1}{6}\frac{r_0''}{r_0^3} - \frac{1}{6}\frac{\Phi_0''}{r_0^2} \\ 4\frac{r_0''/r_0}{\epsilon^2} + 2\frac{\Phi_0''}{\epsilon^2} - \frac{1}{6}\frac{r_0''}{r_0^3} - \frac{1}{6}\frac{\Phi_0''}{r_0^2} \\ -2\frac{r_0''/r_0}{\epsilon^2} \end{pmatrix} + \begin{pmatrix} -\frac{4}{3}\frac{\Phi_0''}{\epsilon^2} + \frac{1}{9}\frac{\Phi_0''}{r_0^2} \\ -4\frac{\Phi_0''}{\epsilon^2} + \frac{1}{9}\frac{\Phi_0''}{r_0^2} \\ \frac{4}{3}\frac{\Phi_0''}{\epsilon^2} \end{pmatrix}
$$
$$
+ \begin{pmatrix} \frac{2}{3}\frac{r_0''/r_0}{\epsilon^2} - \frac{2}{3}\frac{\Phi_0''}{\epsilon^2} - \frac{1}{18}\frac{r_0''}{r_0^3} + \frac{1}{18}\frac{\Phi_0''}{r_0^2} \\ -\frac{8}{3}\frac{r_0''/r_0}{\epsilon^2} + \frac{8}{3}\frac{\Phi_0''}{\epsilon^2} \\ +\frac{5}{3}\frac{r_0''/r_0}{\epsilon^2} - \frac{5}{3}\frac{\Phi_0''}{\epsilon^2} - \frac{1}{36}\frac{r_0''}{r_0^3} + \frac{1}{36}\frac{\Phi_0''}{r_0^2} \end{pmatrix}. \tag{31}
$$

With the same accuracy we get for components of Riemannian tensor at the throat (up to index permutations and $\hat\theta \leftrightarrow \hat\phi$):

$$
R_{\hat\theta\hat\phi\hat\theta\hat\phi} = r_0^{-2}, \quad R_{\hat\rho\hat\theta\hat\rho\hat\theta} = -\frac{r_0''}{r_0}, \quad R_{\hat t\hat\rho\hat t\hat\rho} = \Phi_0''. \tag{32}
$$

Others are zero in this approximation. Using Christensen's formula for $T^{\mu\nu}_{\rm div}$ with this input and subtracting it from $T^{\mu\nu}_{\rm reg}$ cancels all the singularities, as it should; besides that, we have for correction to finite part:

$$
2\pi^2 \begin{pmatrix} T_{\hat t\hat t} \\ T_{\hat\rho\hat\rho} \\ T_{\hat\theta\hat\theta} \end{pmatrix}^{(1)}_{\rm div,finite} = \begin{pmatrix} -\frac{1}{12}\frac{r_0''}{r_0^3} \\ -\frac{1}{6}\frac{r_0''}{r_0^3} - \frac{1}{18}\frac{\Phi_0''}{r_0^2} \\ -\frac{1}{12}\frac{r_0''}{r_0^3} - \frac{13}{72}\frac{\Phi_0''}{r_0^2} \end{pmatrix} \tag{33}
$$

With taking into account all the contributions we ultimately get for total renormalized tensor in the considered approximation:

$$
2\pi^2 \begin{pmatrix} T_{\hat t\hat t} \\ T_{\hat\rho\hat\rho} \\ T_{\hat\theta\hat\theta} \end{pmatrix}^{(0)+(1)}_{\rm ren} = \begin{pmatrix} \frac{1}{60 r_0^4}\ln\frac{\Lambda}{L} - \frac{5}{36}\frac{r_0''}{r_0^3} \\ -\frac{1}{60 r_0^4}\ln\frac{\Lambda}{L} \\ \frac{1}{60 r_0^4}\left(\ln\frac{\Lambda}{L} + \frac{1}{2}\right) - \frac{1}{36}\frac{r_0''}{r_0^3} + \frac{5}{72}\frac{\Phi_0''}{r_0^2} \end{pmatrix}. \tag{34}
$$

3.Discussion. Taking the difference between radial pressure $\tau = -T_{\hat\rho\hat\rho}$ and energy density $\varrho = T_{\hat t\hat t}$ obtained we find it to be positive at the throat:

$$
\tau - \varrho = \frac{5}{72\pi^2}\frac{r_0''}{r_0^3} > 0. \tag{35}
$$

This is quite nontrivial result, since generally we might get arbitrary combination of Φ_0'', r_0''/r_0, not necessarily simply r_0''/r_0 with positive coefficient. Thus, WEC is violated at the throat, at least on the level of second derivatives: not only LHS of gravity eqs. (Einstein tensor) implies $\tau > \varrho$, but also this condition is fulfilled by RHS (electromagnetic stress-energy tensor) for given topology. As for the derivatives of larger order, their contribution to $\tau - \varrho$ is not a'priori fixed in sign, but the fact that required sign is achieved already on the level of second derivatives, means that contribution of higher derivatives required for further adjustment of given value need not be larger then that of first ones, which is normal condition of applicability of perturbation theory and regularity of possible solution.

Large logarithms do not contribute $\tau - \varrho$ in the given approximation. This is because corrections to the structure

$$g^{-1/2}\frac{\delta}{\delta g_{\mu\nu}}\int C_{\mu\nu\lambda\omega}C^{\mu\nu\lambda\omega}g^{1/2}d^4x \tag{36}$$

entering $T^{\mu\nu}$ as coefficient at logarithm, prove to occur only at the level of four differentiations. To see effect of the fourth derivatives we linearly expand (36). Einstein eqs. at the throat read:

$$\begin{aligned}
R_{\hat{t}\hat{t}} - \frac{1}{2}Rg_{\hat{t}\hat{t}} &= \frac{1}{r_0^2} - 2\frac{r_0''}{r_0} = \frac{\kappa}{2\pi^2}\left\{\frac{N}{60r_0^4}\left[1 + 2r_0^4\left(\Phi_0^{iv} - \frac{r_0^{iv}}{r_0}\right)\right] - \frac{5}{36}\frac{r_0''}{r_0^3}\right\},\\
R_{\hat{\rho}\hat{\rho}} - \frac{1}{2}Rg_{\hat{\rho}\hat{\rho}} &= -\frac{1}{r_0^2} \qquad = \frac{\kappa}{2\pi^2}\left(-\frac{N}{60r_0^4}\right),\\
R_{\hat{\theta}\hat{\theta}} - \frac{1}{2}Rg_{\hat{\theta}\hat{\theta}} &= \Phi_0'' + \frac{r_0''}{r_0}\\
&= \frac{\kappa}{2\pi^2}\left\{\frac{N}{60r_0^4}\left[1 + r_0^4\left(\Phi_0^{iv} - \frac{r_0^{iv}}{r_0}\right)\right] + \frac{1}{120r_0^4} - \frac{1}{36}\frac{r_0''}{r_0^3} + \frac{5}{72}\frac{\Phi_0''}{r_0^2}\right\}.
\end{aligned} \tag{37}$$

Here $N = \ln\frac{\Lambda}{L}$, $\kappa = 8\pi G$, G is Newtonian gravity constant. Not shown are bilinear in second derivatives terms multiplying N which in precise calculations should be taken into account on equal footing with fourth derivatives. From $\rho\rho$ - equation the estimate for wormhole radius follows,

$$r_0^2 = \frac{\kappa}{240\pi^2}\ln\left(\frac{240\pi^2}{\kappa}\frac{\Lambda^2}{k^2}\right) \tag{38}$$

(we have taken into account that N itself depends on r_0 through $L = kr_0$, $k \sim 1$). Then it follows from two other eqs. that

$$\Phi_0'', \frac{r_0''}{r_0} \sim \frac{1}{r_0^2}, \quad \Phi_0^{iv}, \frac{r_0^{iv}}{r_0} \sim \frac{1}{r_0^4}. \tag{39}$$

In other words, the length of the wormhole is of the order of it's radius. Thus, approximation of long wormhole seems to be not relevant to selfconsistent solution, but we can hope to approach qualitatively physical wormhole starting from the sufficiently long one.

As for the range of possible values of r_0, it follows from the experimental limitations on Λ. The latter arise in connection with that the induced C^2-term should violate experimentally observed planetary motion, especially that of mercury [5]. The C^2-term enters action with coefficient just of the order of r_0^2/κ (the r_0^2 itself as given by (38) is defined by this coefficient), i.e. with coefficient $\sim r_0^2$ relative to Einstein R-term. So the effect of C^2-term has relative value like $\exp(-r_{\rm merc}/r_0)$ [5]. Here $r_{\rm merc}$ is typical radius of mercury orbit. So the requirement for this correction not to exceed experimental accuracy leads to r_0 being at least 1.5 orders of magnitude smaller than the radius of mercury orbit, i.e. of the order of the radius of sun. As for the lower limit for r_0, it can be estimated by taking $\Lambda \sim 10^{62}$ in Plank units, which corresponds to experimentally observable part of Universe.

Thus, the range of experimentally acceptable (in indirect way) values of r_0 is quite large: it can be by half an order smaller than the Plank length (though in this region the concept of classical gravity may be wrong) and it can be as large as radius of sun.

Finally, these estimates do not take into account the influence of another fields, of which the most essential may be that of massless particles (e.g. the neutrino one) due to occurrence of large logarithms. Corresponding work is in progress now.

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